SPATIAL INTERACTIONS AND REGIONALISATION OF THE VYSOČINA REGION USING THE GRAVITY MODELS

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Abstract
The submitted study is primarily aimed at the application of the gravity models to the research of theoretical and real spatial interactions and the theoretical and real regionalisation of the Vysočina Region in the Czech Republic. The interactions among the regional centres and the centres and their hinterland are a key component of the geographical organization of an area. The Reilly's law belongs to the basic gravitation models used in the geographical research, especially for studying the interaction patterns among the settlement elements. Despite the fact that the gravitation models were used in the human geography especially in the second half of the 20th century, they still form an important part of the geographical methodology originating from the social physics. The outcome of the study is comparison of the real and theoretical interactions in the Vysočina Region and the study of relevance of the Reilly's model application under specific geographical conditions.

Key words: spatial interactions, gravity models, regionalisation, Reilly's law, Vysočina region.

INTRODUCTION

The geographical area is by its nature considerably heterogeneous. Its internal heterogeneity especially arises from the diversity and variability of positioning of the settlement elements (establishments, networks, areas) which co-exist and intercommunicate upon the differently intensive spatial interactions. It is just the intensity of spatial interactions among these elements which is a key indicator of their mutual complementarity. The interactions of various availability and intensity give rise to the links that are frequently used in the human geography for defining nodal regions. The nodal regions can be defined in this context as territorially confined portions of the earth surface being formed on the basis of interactions between the settlement centres and their hinterlands by way of functional relations (Klapka, Tonev 2008). Creation of nodal regions is an important part of geographical research, as the intensity of spatial interactions reflects the substantial features of the society's spatial organization (for more details please refer to the studies by Haggett 2001 or Morrill 1974). The most often used sources in spatial interactions include especially the migration flows, commuting flows, transport connections, etc. Given their unique nature, the widely interpreted spatial interactions (movement of people, freight, information and capital) thus became one of the fundamental concepts in geographical research in the second half of the 20th century in connection with the so-called quantitative revolution in the human geography (see the discussion in the study by Ullman 1980). This revolution gave rise to the massive development of mathematized approaches to the study of spatial interactions geared towards the application of gravitation models because the gravitation models in many respects managed to replace the missing real data on spatial interactions between various geographic locations. This resulted in the increased number of studies which methodologically relied on the application of gravitation models in the transport and geographic research, among others (for more details please refer to the studies by Hay 2000 or Řehák 1988). The Reilly's law has become one of the most frequently used gravitation model, as it not only managed to determine the size of spatial interactions between the geographic locations on the basis of empiric calculations, but also to search for the limits of powers of individual centres and, therefore, it became an appropriate tool for regionalisation of the territory. This is why the Reilly's law became one of the first forms of application derived from the gravitation model in geography (Hlavička 1993). Later on, the quantitatively oriented approaches towards the study of spatial interactions were criticised for their excessive descriptiveness and little focus on their social essence. However, even today the gravitation models belong to the frequently used research tools in the human geography, especially thanks to their simplicity and declared capability of theoretic expression of real relations (Wilson and Bennet 1985).

The presented study is thus aimed to verify and critically evaluate the Reilly's law application to the research of spatial interactions and the geographical area regionalisation using the Vysočina Region, Czech Republic, as an example (initial demarcation from the 2001). The defined theoretical interactions among the Vysočina Region's settlement centres are compared with real interactions based on especially the commuting and transport connections. The second part is concerned with the comparison of theoretical regionalisation of the Vysočina Region upon determining a so-called equilibrium point among the settlement centres according to the Reilly's law and the real regionalisation of the monitored territory on the basis of defining the nodal regions using the intensity of commuting to work.

REILLY'S LAW AND ITS APPLICATION IN HUMAN-GEOGRAPHICAL RESEARCH

Originally, the Reilly's law was designed on the service geography platform, as it was initially used to model the catchment areas of inhabitants upon the attraction by individual types of services (Reilly 1931). The model was based on the Newton's physical law which states that any two objects exert a gravitational force of attraction on each other and the magnitude of the force is proportional to the gravitational masses of the objects and inversely proportional to the distance between them. This fact was used by Reilly in modelling the catchment areas upon attraction by services, in which he observed some regularities to paraphrase the Newton's law, that the force of interaction between two towns is proportional to the size (e.g. population) of the towns and inversely proportional to the increasing distance between them. This principle was later modified by the Huff's law (Huff 1963), upon which the probability of the inclination, the size of centres expressed by the selling area and the time accessibility of individual settlement centres were newly incorporated in this relation. The basis of the Reilly's law was to the determine probability of the service-driven inclination of one place in relation to one of two accessible service centres (for more details please refer to the study by Maryáš 1983). This fact was often criticized by various authors (Berry 1967), as it lacked the choice of more than two centre options which would have made the model real. The study by Řehák et al. (2009), however, argues that this criticism is not justified, since if one can make a choice of three or more catchment centres, these centres may be viewed gradually by individual pairs. The study by Klapka et al. (2010) mentions that the theoretical and methodical fundamentals of the interaction model is predominantly formed by its mathematical expression, various model forms, conditions under which the model may be used, its limitations, potential drawbacks and issues related with the model use. The basic formula for calculation of theoretical interaction between two settlement centres is as follows:

$$I_{AB} = \frac{M_A \cdot M_B}{d_{AB}} \quad [1]$$

where M_A and M_B are masses (e.g. population numbers) of the settlement centres and d_{AB} is the distance between them. The Reilly's law transforms this basic form of the gravitation model into the relationship in which not only theoretical spatial interactions but also the interface between the influences of two centres is determined. According to the study by Halás, Klapka (2010), the Reilly's law principle is determined by the line interface between the spheres of influence in the pair of centres. The model can also be correctly applied to three and more centres and the centre with a dominant influence can be clearly determined for each point in the territory. Originally, the Reilly's law was a mere application of the gravitation law, with each centre having its own weight. The interface is fixed by the coefficient ***k*** to be calculated as follows:

$$k = \sqrt{\frac{M_A}{M_B}} = \frac{d_{AB} - n}{n} \qquad [2]$$

where M_A and M_B are the weights (or masses) of two centres being compared (generally, it is used in the form where $M_A \geq M_B$). Practically, it means that the limit between the spheres of influence of two centres is the set of points the distance of which from the centre A is the ***k***-multiple of their distance *n* from the centre B.

The study by Řehák et al. (2009) also mentions that there are the following three versions of the Reilly's law:

- **a) Geometric version** is the simplest model version where the influences between two pairs of the strongest centres are always monitored as well as the interactions between the third strongest centre and the stronger centre in the half-plane of which such centre is located, etc. The geometric version of the Reilly's law was used, e.g. in the study by Halás, Klapka (2010), in which the territory of the Czech Republic was divided on the basis of circular arcs drawn around all centres with population of more than 25 thousand.
- **b) Topographic version** especially differs from the above one in that it considers the specific conditions of the territory (e.g. transport network) and is, generally, deemed to be more exact. The study by Řehák et al. (2009) also mentions that the elimination method (play off) is employed in the topographic version to determine the pertinence of units (e.g. municipalities) to the centres. In the Czech Republic, it was used in the study by Kladivo et al. (2010) and in Slovakia in the study by Halás (2005), for instance.
- **c) Oscillating version** – represents the method which was first mentioned in the study by Řehák et al. (2009). It is a certain modification of the topographic version the results of which provide absolute regionalisation of the geographic area, however, they fail to determine "intermediate zones" in which the dominance of relevant centres may be lower or, more precisely, less clear.

RESEARCH METHODS

The first research phase was focused on monitoring the theoretical and spatial interactions between the settlement centres (centres of municipalities with a designated community authority). The monitored centres included: Jihlava, Telč, Třebíč, Moravské Budějovice, Náměšť nad Oslavou, Žďár nad Sázavou, Velké Meziříčí, Bystřice nad Pernštejnem, Nové Město na Moravě, Havlíčkův Brod, Chotěboř, Světlá nad Sázavou, Pelhřimov, Humpolec and Pacov. Their regional powers within the Vysočina Region are clear, since even the study by Hampl (2005) considers them to be the region's independent centres of at least subregional importance. There were also monitored centres (neighbouring municipalities with extended power) outside the Vysočina region for more complexity of the research. The centres outside the Vysočina region included: Jindřichův Hradec, Dačice, Tábor, Vlašim, Čáslav, Chrudim, Hlinsko, Polička, Tišnov, Rosice, Ivančice, Moravský Krumlov, Znojmo and Boskovice.

The theoretical spatial interactions among these centres were monitored upon their population on 1 January 2011 and the distance among them was measured in the road network. The theoretical interactions were monitored using the simple version of gravitation model in the following form (see equation 1), upon which the value of a dimensionless number representing their theoretical interaction was achieved. For instance, the value of the Jihlava–Havlíčkův Brod relation, calculated using the above formula, amounted to 52,828.53. The matrix of individual relations was then relativized and finally expressed using the relative shares of force through interactions between individual pairs of centres. For the above relation, the value of 7.46 per cent of all interactions in the monitored group of centres was achieved. The similar method was also used in the study by Řehák (1992), for example.

The real spatial interactions which serve for comparison with theoretical values were formulated using the absolute number of commuters to work between individual centres upon the data obtained from the Population and Housing Census in 2001 (later data is still unavailable) and the bus connection data between the settlement centres obtained from the IDOS electronic timetable database. In the first case, the sum of total of people commuting from the centre A to the centre B and from the centre B to the centre A was monitored. In this way, we obtained the intensity of two-way real interaction which is highly justified in the human geographical research and as such it is very often used to monitor the links between the centres and individual regions (see, for instance, Toušek et al. 2005, Hlavička 1992 or Hampl 2005 for defining the metropolitan regions). In the second case, we obtained the transport interactions that are often used for calibrating the spatial interaction models (e.g. Řehák 1992, Chmelík 2008), using the number of direct two-way bus connections between the centres in one day (Wednesday, 2 November 2011).

The second research phase focused on the comparison of the theoretical and real socially geographical regionalisation of the region. As the initial relation for the model territory regionalisation upon the theoretical interactions, the Reilly's law, or more precisely, its topographic version was used to search for all potential equilibrium points between the competences of two centres according to the formula:

$$n = \frac{d_{AB}}{1 + \sqrt[5]{\frac{M_A}{M_B}}} \qquad [3]$$

where n is the distance to the centre with higher mass, d_{AB} is the distance between two monitored centres, and M_A and M_B are masses of monitored centres ($M_A \geq M_B$). The size (mass) of individual centres was again expressed as a number of permanent residents in all monitored centres on 1 January 2011. The spacing between the monitored municipalities and their potential centres was measured in the road network. For optimum representation of monitored relations, we worked, after testing a few variants, with the fifth root to determine the interface between two centres (similarly, Kladivo et al. 2010). When we used the basic form (i.e. square root), the role of minor centres, or more precisely, the centres with lower population, was often disadvantaged, while, on the other hand, the influence of bigger centres was overestimated. On the contrary, when testing the 7th root coefficient, the importance of minor centres enormously increased and the "natural" importance of minor centres decreased in the regionalisation. Therefore, the variant which seemed to be optimum for the given hierarchical level was chosen. This methodological procedure made it possible to allocate all municipalities within the Vysočina Region to any of the monitored centres, upon the probability of inclination of these municipalities to any of the potential centres. With respect to the foreseen inclination of some villages to the centres outside the Vysočina Region, also the set of centres (municipalities with a designated community authority) situated in the neighbouring regions was included in the analysis so that the results of regionalisation were as exact as possible. In the spirit of the play off method system (see Řehák et al. 2009), a pair of probable centres to which the given municipality may be theoretically inclined was monitored for each municipality. The regionalisation procedure is schematically represented in Figure 1 and in the text below.

As an example of the regionalisation procedure, we can present the calculation of theoretical inclination of the municipality of Stropešín located near Náměšť nad Oslavou, a municipality with extended powers. From the perspective of theoretical inclination of this municipality, the potential centres are Náměšť nad Oslavou and more distant Třebíč. In 2011, the total resident population of Jihlava and

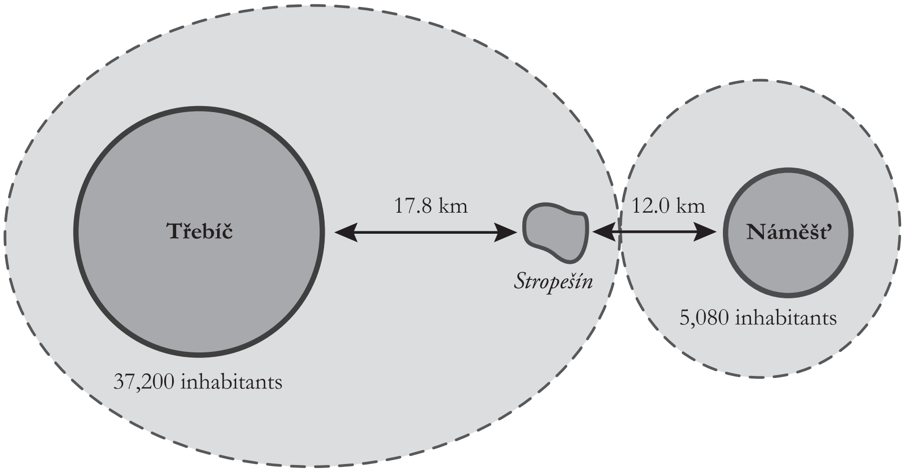

Figure 1 Example of the regionalisation procedure applied to the municipality of Stropešín.
Source: own draft.

Telč amounted to 37,200 and 5,080 respectively. The municipality of Stropešín is 12.0 km far from Náměšť nad Oslavou and 17.8 km far from Třebíč. Using the above formula, it is clear that the maximum regional influence of Náměšť nad Oslavou achieves the distance of 10.2 km. This is less than the real distance between Stropešín and Náměšť nad Oslavou, therefore, the municipality, according to the Reilly's law, inclines to Třebíč.

The real regionalisation of the Vysočina Region was, again, carried out using the data on commuting to work in the Vysočina Region from 2001. The inspiration for the regionalisation procedure based on commuting flows was the study by Novák (2009). With each municipality, we worked with the same pair of settlement centres. Each municipality in the Vysočina Region was allocated to the relevant centre upon the number of residents who were commuting to work in one or the other centre in 2001. Based on this information, it was determined to which centre the given municipality inclined. In case of equal numbers of commuters, the additional indicators, such as direct transport links, centre size and the distance between the centres and municipalities, were taken into account. The number of especially direct transport connections was monitored for direct connections, upon which the municipality was allocated to the centre (the municipality was allocated to the centre with a higher number of direct transport links). In some cases, not a single person commuted from the municipality to the defined centres. In this case, the usual method used was the monitoring of commuting to less significant centres in the given region or allocation on the basis of the second strongest commuting direction (see Halás et al. 2010). For instance, as the potential centres for the municipality of Zvěstonice in the Havlíčkův Brod region, the towns of Havlíčkův Brod and Čáslav (Central Bohemia Region) can be considered. A total of 21 local residents commute to work outside the municipality. Of that 16 commute to the municipalities which belong to the Central Bohemia Region or to the municipalities which incline to Čáslav in the Vysočina Region. As a result, this municipality inclines to the catchment area the centre of which is located outside this region. As far as the commuting to work is concerned, the border municipalities belong to the catchment area of the centres located outside of the Vysočina Region. The commuting centres, basically, do not change – only the number of municipalities which are inclined to them. Unlike the model regions upon the Reilly's law, the only change concerns the Rosice centre attracting one municipality. The municipality of Ivančice, on the other hand, lost this position.

THEORETICAL AND REAL INTERACTIONS IN THE VYSOČINA REGION

Within the calculated theoretical interactions among the monitored centres, the regional town of Jihlava plays a dominant role, naturally forming the region centre, although in the nation-wide context it is only ranked among the centres of microregional importance (see Hampl 2005). The dominant role of Jihlava is also promoted by the central position of the centre in the region and relatively equally distributed larger centres around (namely Třebíč, Havlíčkův Brod and Žďár nad Sázavou). The regional town thus achieves the highest share of all calculated theoretical interactions. The strongest interactions are manifested by the relations Jihlava–Třebíč (8.9%) and Jihlava–Havlíčkův Brod (7.5%). The other, more important flows are located among the centres in the same former districts, which is mainly caused by their small distance in kilometres. This fact is most strikingly demonstrated in the relations Žďár nad Sázavou–Nové Město na Moravě (3.6%) and Havlíčkův Brod–Chotěboř (2.4%). The other more important flows include Třebíč–Velké Meziříčí, Havlíčkův Brod–Žďár nad Sázavou and Třebíč–Nové Město na Moravě. Table 1 lists ten most important theoretical interactions among the monitored relations of centres. The other monitored centres are characterized by the lower theoretical interactions, especially between the non-district settlement centres. Here, their bigger distance and, of course, their low importance given by the lower population is manifested. The weakest interactions are taken especially by the settlement centres Pacov and Náměšť nad Oslavou, located in the peripheral areas of this region. Practically all interactions with Náměšť nad Oslavou do not exceed 0.5 per cent of all calculated theoretical interactions. The only exception is the relation between this centre and Třebíč (former district town) and Jihlava (regional town). The same phenomenon can be seen in case of Pacov where the interaction limit of 0.5 per cent is exceeded only for Jihlava and Pelhřimov (regional town to which Pacov belongs). The weakest interaction was observed especially between Pacov and Náměšť nad Oslavou (0.04 per cent). The matrix of all theoretical interactions is graphically represented in Figure 2.

The real spatial interactions in the Vysočina Region were first monitored by way of presence of direct bus connections. Upon the methodological procedure, the absolute numbers of transport links among the monitored centres were defined and then relativized so that the bus interactions could have been compared as much as possible with the calculated values of theoretical interactions. In this way, the transport links can be perceived as a very important feature and one of the most significant manifestations of spatial interactions. In this context, the study by Řehák (1988) mentions that the transport relations belong to not only key interactions in the geographic area, but also indicate the interactions between the centres and regions especially on the basis of their complementary transport relations.

The results arising out of definition of theoretical spatial interactions among all monitored centres may be practically identified with the results of direct bus connection numbers, especially for the bus interactions with Jihlava. The change occurs in the absolutely highest number of connections, with a dominant role taken by Žďár nad Sázavou (Table 2). For this type of interactions, Jihlava, however, continues to hold a very important position. Most direct connections can be found between Žďár nad Sázavou and Nové Město na Moravě (the share amounts to 9.9%). The other important flows include Telč–Jihlava (7.2%) and Pelhřimov–Humpolec (6.5%). Here, also the higher number of connections among the centres located in the same district is apparent. This fact is more prominent than for theoretical interactions, especially as a result of high rigidity of traffic services organisation within the former districts (this fact is addressed in the studies by Marada et al. 2010 or Kraft 2012, for example). The exception is, however, in the Třebíč district, where there are no direct bus connections between Náměšť nad Oslavou and Moravské Budějovice. The most intensive flows located outside the same district include Jihlava–Velké Meziříčí, Jihlava–Humpolec, Jihlava–Moravské Budějovice and Havlíčkův Brod–Humpolec. The least number of direct bus connections was observed in the Pacov centre – 26 connections leading to three centres only. This is primarily caused by the peripheral location of this

Table 1 Strongest theoretical interactions among monitored centres. Source: own calculations.

Relation	Distance (km)	Share of all interactions (%)
Jihlava–Třebíč	30.3	8.9
Jihlava–Havlíčkův Brod	23.4	7.5
Jihlava–Žďár nad Sázavou	30.8	5.4
Jihlava–Pelhřimov	26.8	4.5
Žďár nad Sázavou–Nové Město na Moravě	9.5	3.6
Třebíč–Velké Meziříčí	17.7	3.5
Jihlava–Humpolec	23.0	3.5
Havlíčkův Brod–Žďár nad Sázavou	26.4	2.7
Jihlava–Velké Meziříčí	31.6	2.7
Havlíčkův Brod–Chotěboř	14.0	2.4

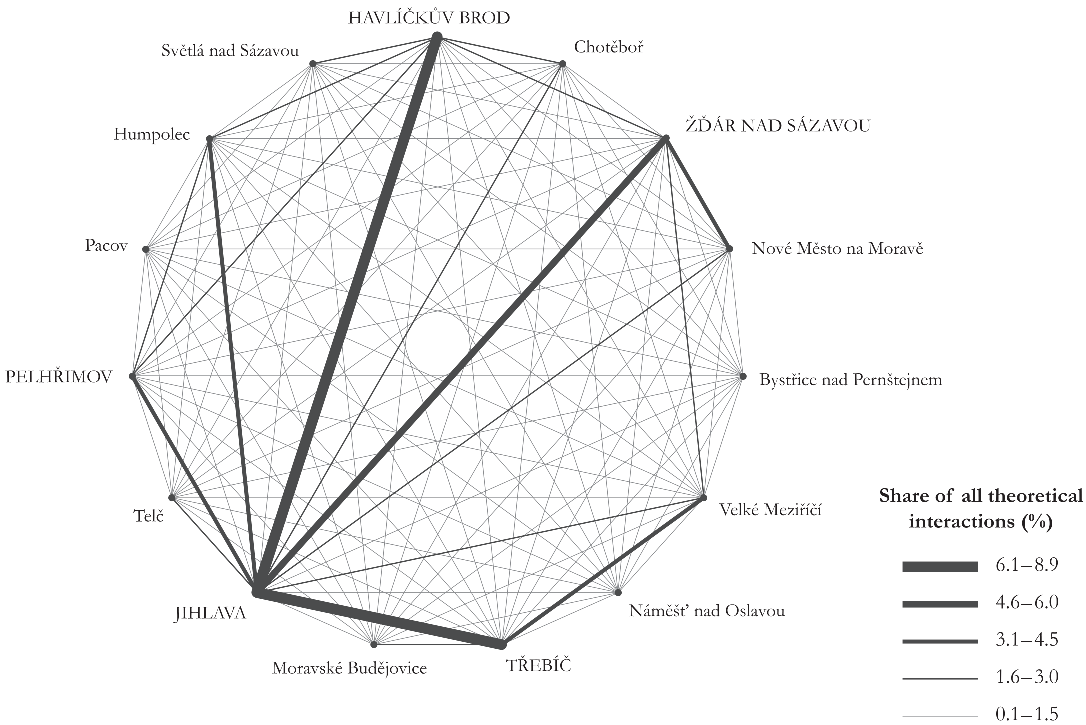

Figure 2 The matrix of all theoretical interactions among monitored centres.
Source: own calculations.

Table 2 Strongest bus interactions among monitored centres.
Source: IDOS 2011/2012, own calculations.

Relation	Number of bus connections	Share of all bus connections (%)
Žďár nad Sázavou–Nové Město na Moravě	96	9.9
Jihlava–Telč	70	7.2
Pelhřimov–Humpolec	63	6.5
Havlíčkův Brod–Chotěboř	48	5.0
Třebíč–Moravské Budějovice	44	4.5
Jihlava–Velké Meziříčí	43	4.4
Jihlava–Žďár nad Sázavou	42	4.3
Havlíčkův Brod–Světlá nad Sázavou	40	4.1
Jihlava–Havlíčkův Brod	39	4.0
Jihlava–Třebíč	37	3.8

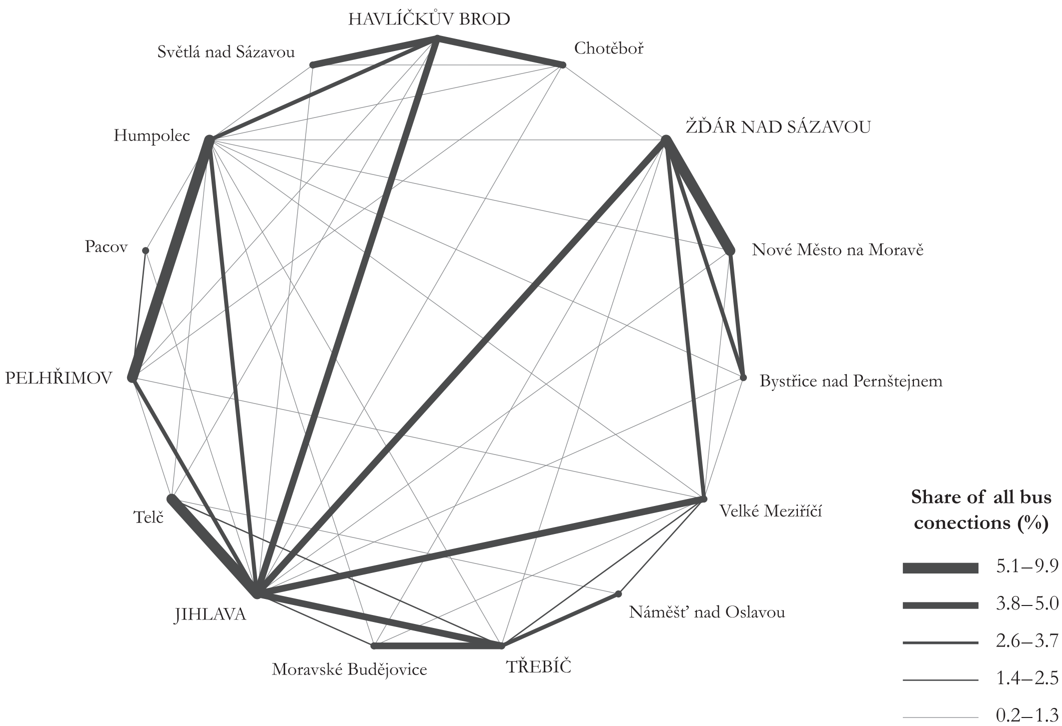

Figure 3 The matrix of all bus interactions among monitored centres.
Source: own calculations.

Table 3 Strongest commuting interactions among monitored centres.
Source: Population and Housing Census 2001, own calculations.

Relation	Number of commuting persons	Share of all commuting connections (%)
Žďár nad Sázavou–Nové Město na Moravě	1,131	19.7
Havlíčkův Brod–Chotěboř	619	10.8
Jihlava–Havlíčkův Brod	375	6.5
Pelhřimov–Humpolec	367	6.4
Třebíč–Náměšť nad Oslavou	349	6.1
Havlíčkův Brod–Světlá nad Sázavou	305	5.3
Jihlava–Třebíč	252	4.4
Nové Město na Moravě–Bystřice nad Pernštejnem	234	4.1
Třebíč–Moravské Budějovice	219	3.8
Pelhřimov–Pacov	210	3.7

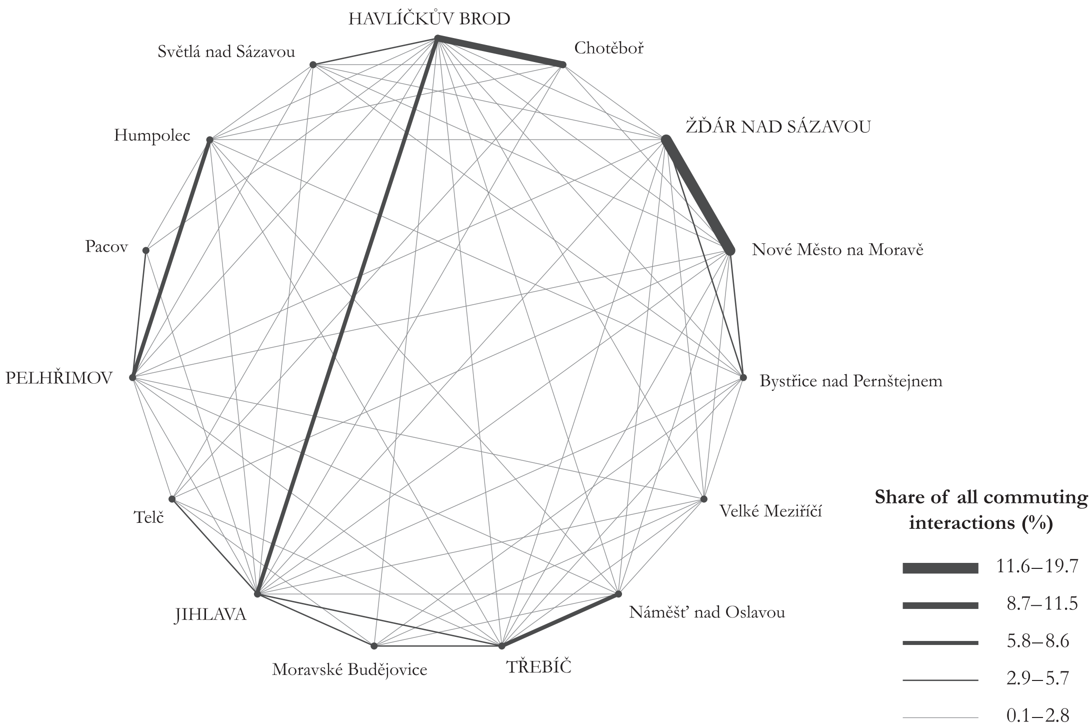

Figure 4 The matrix of all commuting interactions among monitored centres.
Source: own calculations.

centre within the monitored region. The lowest bus interactions can be found in the Náměšť nad Oslavou settlement centre with the direct bus lines with Třebíč and Telč only. The absolutely weakest relations (2 direct connections, 0.2% of the monitored total) of existing direct bus connections are led among the region centres which are farther from each other, since mostly at least one of the pair of centres is located at the farther place in the region. As already noted above, it can be assumed that any possible inclusion of train connections in the matrix of transport links would make the model less clear, especially as a result of train lines unlike the rather nodal arrangement of the bus transport. The matrix of all existing real bus interactions is shown in Figure 3.

The last monitored category was the real interactions represented by way of an absolute number of commuters to work upon the 2001 Population and Housing Census data. Commuting among all monitored centres in the Vysočina Region considerably less imitates the above two types of spatial interactions. The main reason is the persistent closeness of commuting interactions within the former districts, which is frequently referred to in the study by Hampl (2005). As a result, the commuting interactions within the same district are often on the higher level than the interactions among the main settlement centres. Therefore, this is manifested by the microregional nature of everyday commuting processes, being relatively significant in 2001 (for instance, Sýkora, Mulíček 2009). Hence, the dominant role of Jihlava is not as important as in above cases, as most of the flows leading to the regional town are not much different from other important flows in this region. The relations among monitored centres in the same district, again, have the highest percentage share.

The absolutely highest number of commuters (1,131) was observed between the centres Žďár nad Sázavou and Nové Město na Moravě (19.7%, see Table 3 and Figure 4). The prominent role is also the high unemployment in the Žďár nad Sázavou district and the fact that Nové Město na Moravě has gone through the minor decline since 1990s. Despite a certain reduction of job opportunities, the settlement centre Žďár nad Sázavou represented an important micro-region commuting centre even for the area of Nové Město na Moravě. The Havlíčkův Brod–Chotěboř relation (10.8%) is another important flow within the region. The lower share in the total commuting interactions is manifested by the relations Jihlava–Havlíčkův Brod, Pelhřimov – Humpolec and Třebíč–Náměšť nad Oslavou. Also the remaining interactions in the Vysočina region, namely interactions between Havlíčkův Brod and Světlá nad Sázavou and between Jihlava and Třebíč, achieve the relatively high share within the monitored area. Moreover, Jihlava and Havlíčkův Brod exhibit, as the only centres, the presence of commuting interactions with all of the other centres. As in case of the direct bus connection intensity, Pacov with the least number of interaction pairs is characterized by a rather marginal importance. The minimum of existing commuting relations can be observed between the centres far from each other, not located in the same district. There are a total of 13 interaction pairs in the region with only one resident commuting between them. There are also additional 18 pairs with only 2 to 5 inhabitants commuting between them.

THEORETICAL AND REAL REGIONALISATION OF THE VYSOČINA REGION

In the second phase of research, the monitored Vysočina Region was divided into lower subregional units according to regional powers of individual centres. First, the regionalisation of the territory was carried out upon the theoretical spatial interactions by determining the equilibrium points among the individual centres' regional powers using the topographic version of the Reilly's law. This regionalisation was then compared with the real regionalisation of the region according to the intensity of commuting to work. The resulting comparison of both types of regionalisation can considerably highlight some strengths and weaknesses of the Reilly's law application to the human geographic research.

The results of the Vysočina Region regionalisation are shown in Figure 5. As expected, among the biggest regions from the perspective of integrated municipalities and population we can find especially the big centres the regional powers of which are

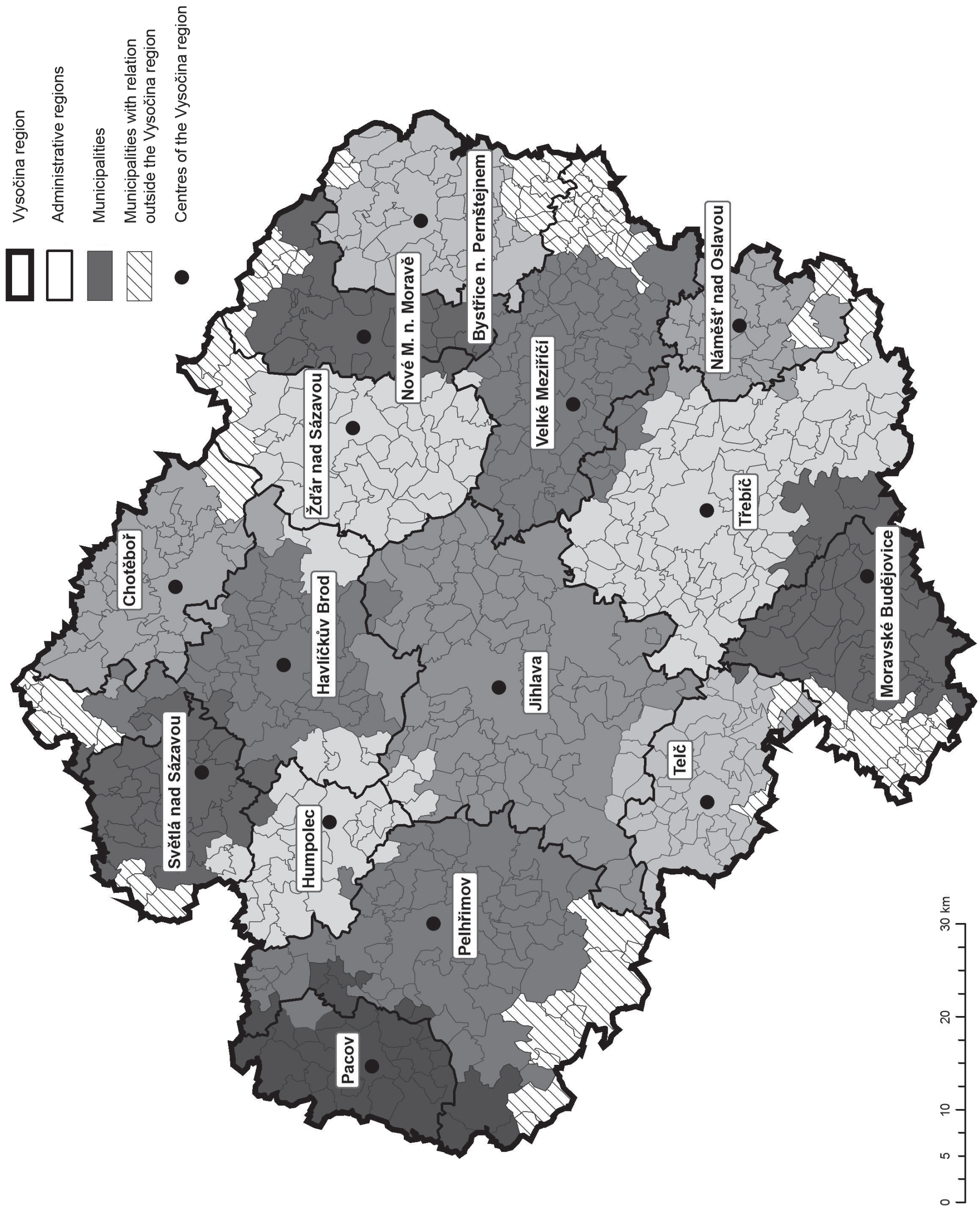

Figure 5 Regionalisation of the Vysočina region based on Reilly's law. Source: own calculations.

Table 4 Basic features of regions delimitated by Reilly's law. Source: own calculations.

Region	Area (km^2)	Number of integrated municipalities	Number of inhabitants
Jihlava	878.4	71	95,689
Třebíč	672.9	81	69,314
Žďár nad Sázavou	442.3	47	44,623
Havlíčkův Brod	375.1	35	37,743
Velké Meziříčí	503.4	63	35,678
Pelhřimov	594.0	53	31,646
Moravské Budějovice	381.5	36	22,934
Chotěboř	338.0	34	22,452
Bystřice nad Pernštejnem	343.3	39	22,390
Světlá nad Sázavou	291.6	32	21,606
Humpolec	321.5	36	20,037
Nové Město na Moravě	254.7	24	18,824
Telč	309.5	47	15,140
Náměšť nad Oslavou	208.4	27	13,945
Pacov	235.8	24	10,364

based on especially their high population potential. From the area and population viewpoint, the biggest regions belong to the centres Jihlava, Třebíč, Žďár nad Sázavou and Havlíčkův Brod (see Table 4). On the contrary, the smallest regions belong to the centres Telč, Náměšť nad Oslavou and Pacov. This fact is primarily caused by the combination of the lower population in centres and the marginal position of centres in the monitored region. What is more, their regional powers are often adversely impacted by the proximity of competitive centres located outside of the Vysočina Region territory. Therefore, a prominent feature of regionalisation is the frequent inclination of some municipalities to the centres located outside the region territory. According to the Reilly's law, of a total number of 707 monitored municipalities, 74 municipalities (10.5%) are in the catchment area of the centres located outside of the Vysočina Region. This phenomenon is most obvious in the administrative regions Velké Meziříčí and Bystřice nad Pernštejnem, where 24 municipalities incline to the Tišnov centre located in the South Moravia Region. It shall be noted here, that the administrative division was reformed in 2005 and a total of 25 municipalities which are basically identical with the above were separated from the Vysočina Region and incorporated in the South Moravia Region. This was caused especially by the more significant trend in inclination to the South Moravia Region (commuting to work, commuting to schools – see below). Similarly, the strong dominant role of the centre located outside of the Vysočina Region is demonstrated in the administrative regions Telč and Moravské Budějovice, where a total of 17 municipalities are in the catchment area of the Dačice centre located in the South Bohemia Region.

Another interesting aspect is also the connection between the regions defined under the Reilly's law and the administrative division of the region in the form of lower regional self-government administrative units, i.e. the catchment districts of the municipalities

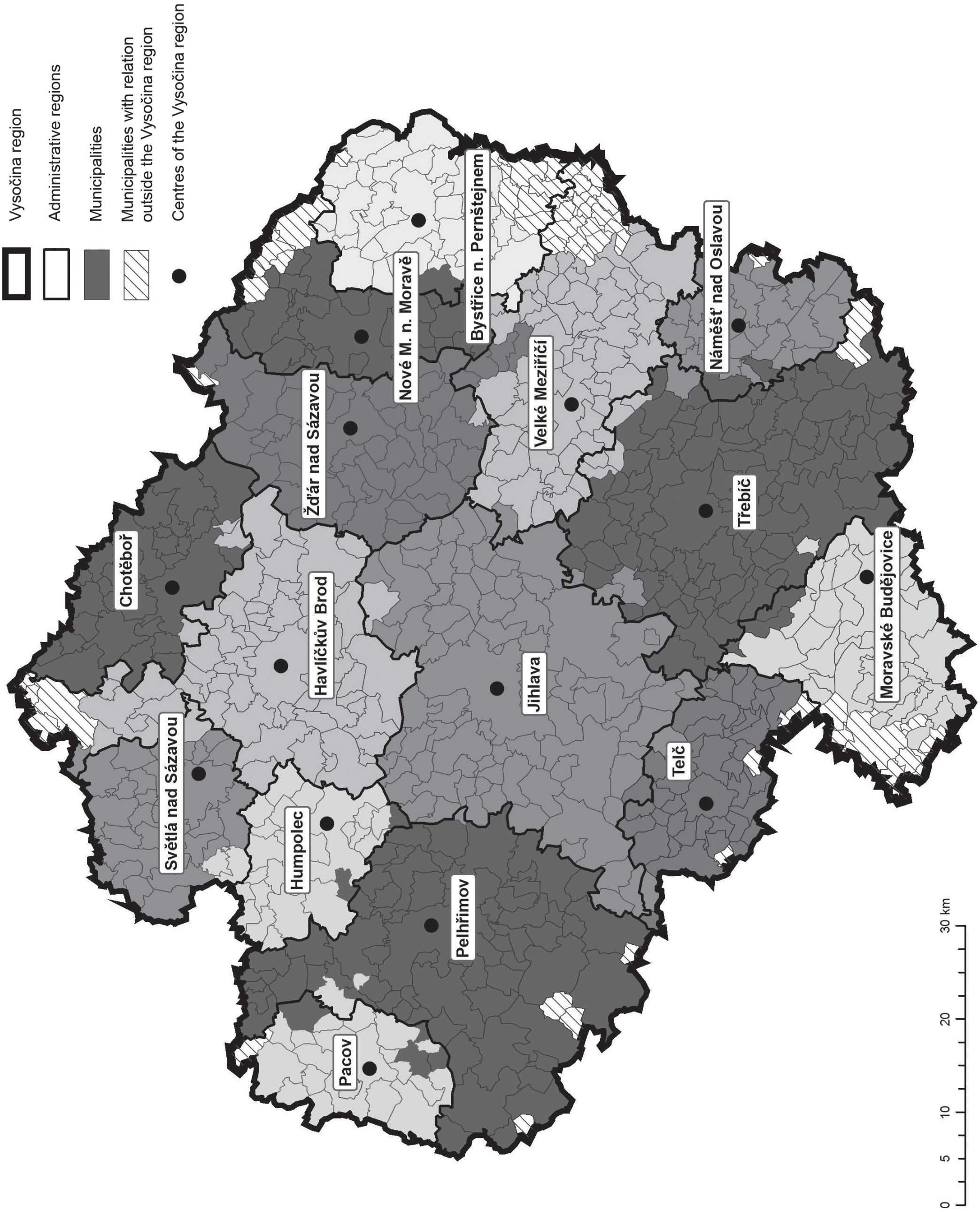

Figure 6 Regionalisation of the Vysočina region based on commuting flows. Source: own calculations.

Table 5 Basic features of regions delimitated by commuting to work.
Source: Populatin and Housing Census 2001, own calculations.

Region	**Area (km^2)**	**Number of integrated municipalities**	**Number of inhabitants**
Jihlava	943.9	84	97,531
Třebíč	798.6	87	75,591
Havlíčkův Brod	625.4	56	49,747
Pelhřimov	829.8	68	45,710
Žďár nad Sázavou	485.1	51	44,667
Velké Meziříčí	466.8	58	35,041
Chotěboř	314.1	28	21,831
Bystřice nad Pernštejnem	318.9	36	21,540
Světlá nad Sázavou	278.9	31	20,778
Moravské Budějovice	317.7	36	18,631
Nové Město na Moravě	259.3	25	18,419
Humpolec	233.5	25	17,039
Telč	262.9	41	13,306
Náměšť nad Oslavou	208.6	26	12,999
Pacov	300.7	19	10,218

with extended powers. The given cartographic representation shows that the theoretical regions are distinctively associated with the administrative division. This is primarily caused by the relatively homogenous distribution of administrative centres in the territory of the region formed especially upon artificial promotion of minor centres in planning the settlement development concept in 1970s. Minor differences are apparent in case of the overlap of the Moravské Budějovice region and the Třebíč municipality with extended powers or Humpolec and the Havlíčkův Brod municipality with extended powers. It shall be, however, noted here that the Reilly's law concept is based on the social physics and is, therefore, rather inappropriate for designing a new administrative division or criticizing the existing one. In this case, much more importance is attributed to the real functional relations generated upon commuting to work, for example. For the Reilly's law review, it is necessary to compare the defined regions with the commuting regions.

Similarly, the commuting regions were defined in the monitored area in line with the above methodological procedure. Whereas the commuting relations reflect substantial features of the spatial organisation of the society (e.g. Klapka et al. 2010), they can be very easily employed for monitoring the relations and evaluating the relevance of the gravitation model application to the geographical research.

The cartographic representation (Figure 6) clearly shows that the regions defined upon the commuting to work are apparently associated with the above regionalisation of the region using the Reilly's law. This is primarily caused by application of the fifth root in the relation, which, as noted above, is probably the most appropriate solution for the evaluated hierarchical level (see Kladivo et al. 2010). Among the other tested variants of the square, cubic and seventh roots, the fifth root appeared to be most representative for this research. As compared with regionalisation of the region according to the commuting relations, however, there are certain differences that point to some disputable characteristics of the Reilly's law. It is especially necessary to highlight that the centres' attractiveness determining their ability to create a sufficiently large catchment area around was, according to the Reilly's law, represented using the number of permanent residents living in the centres, while for the evaluation of the centres' attractivity from the perspective of commuting to work, this relationship includes relatively more factors – especially the centre attractiveness from the perspective of job offer versus the job offer in the centre's hinterland, the labour market situation in the whole region, the level of transport connections, etc. This fact then gives rise to some minor differences in both types of regionalisations of the region. On the other hand, it shall be noted that there is considerable conformity between both types of regionalisation. In the event of regionalisation by the commuting relations, also distinctive closeness of commuting relations is apparent on the microregional level, especially within the districts of municipalities with extended powers and former districts, as frequently referred to in the study by Hampl (2005). The overlaps of administrative boundaries of relevant districts of municipalities with extended powers are minimal in this case, even with the strongest centre from the perspective of commuters to work in Jihlava, which again proves a relatively weak position of this centre within the regional system and its justified ranking among the microregional centres. Similarly, the minor overlaps, or more precisely, inclination of individual municipalities to the centres outside the region's territory, are apparent here. This is clear in the municipalities in the eastern part of the region only (municipalities with extended powers Velké Meziříčí and Bystřice nad Pernštejnem), which were incorporated in the South Moravia Region in 2005.

Also the comparison of features and the hierarchical position of individual regions is very similar for both types of regionalisation (Table 5). A dominant role is, again, played by the Jihlava region with population of more than 97 thousand and 84 integrated municipalities. Unlike the theoretical regionalisation, the number of municipalities increased, however, these municipalities have very low population. Among the centres, which as opposed to the theoretical regionalisation, reinforced their importance, we can name especially Havlíčkův Brod, which reinforced its importance by integration of farther municipalities with higher population in its administrative region, and Pelhřimov which is more important in the commuting relations in the southern part of its administrative region than the theoretically calculated centre Jindřichův Hradec, located in the South Bohemian region. On the other hand, minor centres, such as Telč, Humpolec and Moravské Budějovice, are less important centres in the commuting relations than as expected by the Reilly's law, which is a result of the above criticism of determining the centre attractiveness by the resident population. Consideration could be thus given to the idea of substituting another value in the centre mass – for instance, the work importance of the centre or any other more complex indicator. Its empiric review thus remains a challenge for further research.

CONCLUSION

The achieved results prove that the application of the gravity models to the research of spatial interactions and the regionalisation of geographic area has its justification. It is, in particular, necessary to highlight the benefits of the Reilly's law, or more precisely, of all gravitation models – they make it possible to replace the often missing real data on spatial relations. Hence, it can be an important tool which can be broadly applied not only in the basic research field, but also in the variety of implications for the applied research. To begin with, it is necessary to mention the often used gravitation models for simulating, optimizing and predicting of transport phenomena (for more details please see the study by Řehák 1992 or Halás 2005). However, the combinations of the real data on spatial relations and the calculated theoretical values appear to be more efficient, as they can be better calibrated and adapted to the real situation.

The empirical part of the research demonstrated, among other things, that there is a relatively strong relationship between theoretical and real interactions and at the same time real and theoretical regionalisation of the model area. By the comparison, we can assume that the interactions identified by bus interaction are more independent on the attractivity of individual centers, while the commuting interactions correspond rather to mutual distances of the centers. Gravity models can also be used for detecting the patterns of organization of various human-geographical phenomena. In this context, we can also mention the problematic determination of the exponent value (root value) in the Reilly's law which becomes considerably differentiated when another exponent is used. In this case, the fifth root was used, since it appeared to be the most appropriate root for the regionalisation research on the region level. Use of the lower exponent in the relation would have suppressed the role of minor centres at the expense of larger ones, and on the contrary, the higher exponent would have resulted in inadequate increase of the influence of minor centres and artificial reduction of the larger centres' influence. This reveals one of the biggest weaknesses of the Reilly's law, as the determination of appropriate exponent is always the researcher's task and tends to be subjective to a certain degree. On the other hand, however, it is necessary to note that the application of any model in the research concerning the modelling of complex social systems always represents a certain degree of generalization which cannot be avoided. Therefore, the Reilly's law in the regional geographic research should be viewed as an appropriate tool rather than an absolute result of the entire research process.

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Résumé

Prostorové interakce a regionalizace kraje Vysočina s využitím gravitačních modelů

Hlavním cílem předložené studie je aplikace gravitačních modelů do výzkumu teoretických a reálných prostorových interakcí a teoretické a reálné regionalizace kraje Vysočina v České republice. Vzájemné interakce mezi centry kraje a centry a jejich zázemím jsou klíčovým komponentem a indikátorem geografické organizace prostoru. Jedním z využívaných modelů je Reillyho model patřící mezi základní gravitační modely využívané v geografických výzkumech, především pro studium interakčních vzorců mezi prvky osídlení. Navzdory tomu, že gravitační modely byly v humánní geografii využívány zejména ve druhé polovině 20. století, tvoří dodnes významnou část geografické metodologie mající svůj původ v sociální fyzice.

V příspěvku jsou sledovány teoretické prostorové interakce a jejich vztah k reálným interakcím mezi středisky osídlení (centra ORP) v kraji Vysočina. Teoretické prostorové interakce v kraji Vysočina byly sledovány pomocí jednoduché verze gravitačního modelu na základě vztahu o vzájemné vzdálenosti dvou středisek a jejich populační velikosti. Reálné prostorové interakce byly sledovány pomocí počtu přímých autobusových spojů mezi sledovanými centry z roku 2011 a počtu denně dojíždějících osob na základě dat ze Sčítání lidu, domů a bytů z roku 2001 (novější údaje nejsou dosud k dispozici). Ve druhé části je pak sledována teoretická regionalizace kraje Vysočina na základě topografické verze Reillyho modelu s aplikací páté odmocniny v exponentu vztahu (viz Řehák et al. 2009) a reálná regionalizace kraje na základě dat o dojížďce za prací ze Sčítání lidu, domů a bytů z roku 2001. Vymezené interakce a regiony mohou vhodným nástrojem pro hodnocení relevance použití gravitačních modelů ve výzkumu sídelního systému kraje Vysočina.

V případě hodnocení teoretických a reálných interakcí mezi středisky osídlení bylo prokázáno, že teoretické interakce vymezené na základě prosté verze gravitačního modelu jsou v jisté asociaci s oběma typy reálných interakcí. Větší podobnost vykazují teoretické interakce s interakcemi generovanými přímými autobusovými spoji, neboť přímé („expresní") autobusové spoje často spojují navzájem právě větší centra střediska osídlení. Dá se proto předpokládat, že organizaci autobusových linek kromě jiného ovlivňuje více atraktivita jednotlivých center (v tomto případě počet obyvatel) než jejich vzájemná vzdálenost. V některých sledovaných relacích se však na celkové diferenci teoretických a reálných autobusových spojů podílí více faktorů, z nichž lze zmínit relativně méně frekventovanou železniční dopravu (např. Jihlava–Telč) nebo naopak pozici středisek v rámci sítě

autobusových spojů (například srovnání nejsilnější teoretické interakce Jihlava–Třebíč a jejich relativně slabého autobusového propojení). Organizace dojížďkových interakcí mezi sledovanými centry vykazuje s teoretickými interakcemi relativně menší podobnost. Intenzivní dojížďkové vztahy jsou totiž ovlivňovány větším množstvím faktorů (situace na trhu práce, kvalita dopravního spojení) a z výsledků vyplývá, že spíše než velikost center se na intenzitě vazeb podílí jejich vzájemná vzdálenost (např. srovnání relace Žďár nad Sázavou–Nové Město na Moravě a relace Jihlava–Třebíč).

Vymezené regiony kraje Vysočina na základě Reillyho modelu a jejich srovnání s regiony dojížďky za prací naopak vykazují relativně vysokou souhlasnost. Dojížďkové vztahy bývají zpravidla uzavřenější v administrativních hranicích jednotlivých obvodů ORP, nicméně ani přesahy u vnitřních hranic regionu v rámci teoretických vztahů nejsou nijak markantní. Obě dvě metody regionalizace území však téměř shodně poukazují na relativně slabé vztahy obcí se svými administrativními středisky u vnějších hranic regionu (například ORP Bystřice nad Pernštejnem, Velké Meziříčí a Moravské Budějovice), kde se z hlediska spádovosti v teoretických i reálných dojížďkových vztazích výrazně uplatňují okolní střediska ležící mimo kraj Vysočina. Teoretická i reálná regionalizace mimo jiné rovněž poukazuje na výraznou orientaci vazeb obcí východní části kraje Vysočina (vybrané obce ORP Bystřice nad Pernštejnem), které v roce 2005 přešly z kraje Vysočina do kraje Jihomoravského.