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Spatial influence of regional centres of Slovakia: analysis based on the distance-decay function

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Abstract The uneven distribution of centres in settlement systems and the non-homogeneity of social-economic space are the stimuli for the existence of spatial interactions. The interactions, that manifest the changes in their intensity with an increasing distance from a centre, can be described by distance-decay functions. This paper presents the construction, analysis and typology of distance-decay functions for regional centres of Slovakia using the daily travel-to-work flow data. Apart from an estimation of individual distance decay functions for each centre, a universal distance-decay function is also constructed through more sophisticated statistical analyses, where not only distance is the input parameter, but also the population of a centre. The resulting distance-decay functions have a wide range of uses in spatial interaction modelling (commuting, transportation, etc.). They also define the range of spatial influence of regional centres and therefore they can be used, for example, in proposals and revisions for the administrative division of a territory.

Keywords Spatial interactions · Travel-to-work flows · Distance-decay function · Spatial influence · Radius of influence · Slovakia

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1 Introduction

The space that surrounds us is not homogeneous and its constituent elements are distributed in an uneven manner. The uneven distribution of settlements and the non-homogeneity of socio-economic environments act as stimuli for the existence of spatial flows. Horizontal flows (e.g. of persons, goods, finances, information etc.) form an inseparable part of a social or economic environment and in scientific literature, they are most frequently denoted as spatial interactions. They are understood to be the aggregation of individual mobilities or contacts, which, in the case of population flows, are conditioned by the activities of individuals. Their spatial behaviour is influenced by individuals' needs and their efforts to optimise their spatial movements (or spatial localisation) to gain economic and social benefits. Socio-economic spatial interactions significantly influence the geographical organisation of a society and express the mutual interdependence between regions at various hierarchical levels. The intensity of spatial interactions is differentiated by the location, distance and direction of the interaction. In cases where nodal links are directed at one centre the interaction intensity usually decreases with increasing distance from this centre (of course, minor exceptions can be identified). Much more complex problems arise in the quantification of the decrease (or decay). A distance-decay function can be used for this purpose and it is able to mediate the decrease empirically or graphically with certain accuracy.

The main objective of the paper is to estimate distance-decay functions for (nodal) daily movements of the population to regional centres of Slovakia. Daily travel-to-work data from the population census are used and the interaction decrease will be analysed using several methods with different levels of generalisation. Each centre has its specifics and daily movements in its hinterland are determined by its location, the presence of competing centres and the distance between them, and historical developments. First, the adequacy of two compound distance-decay functions (power-exponential function and Richards' function) is tested and individual distance-decay functions for selected regional centres of Slovakia are estimated with as much accuracy as possible. Second, the universal distance-decay function for all selected regional centres is estimated (with a lower accuracy level) on the basis of the most universal quantitative parameter for centres, i.e. their population size. In this section, the procedure is analogical to be comparable to the procedure published by Halás et al. (2014). Apart from the testing of two individual functions the typology of regional centres of Slovakia according to the distance-decay functions is presented and the relation between spatial influence of regional centres and their location is sought.

2 Theoretical foundations

2.1 Regional and settlement system of Slovakia

At a macro level, the regional and settlement structure of Slovakia is influenced by the elongated shape of the territory in a west-eastern direction. This shape brings increased demands for transport in the direction of its geographical length. This fact was raised during the negotiations over the location of the southern border of Slovakia, and on the basis of the 1920 Trianon peace treaty the new state (then a part of Czechoslovakia) gained control over a considerable part of its present day southern territory (the Lučenec–Košice depression with railway and road connection between Lučenec and Rožňava). Based on its natural potential Lukniš divided the territory of Slovakia into four natural regions: West Slovakian and East Slovakian centralising regions, and North Slovakian and South Slovakian corridor regions (Lukniš 1985: 140–141). Centralising regions represent two individual core areas, where two metropolitan cities, Bratislava and Košice, have gradually formed. Corridor regions connect both core areas and they are divided from each other by the distinct (central Slovakian communication) mountain barrier.

The economic development and current economic level of the North and South Slovakian corridor regions are diametrically different. Until now, an adequate connection between Bratislava and Košice has not been implemented even though the distance is shorter (circa 400 km unlike 460 km along the northern connection). At the same time, there is a lower elevation to be overcome (the Soroška pass is 450 m lower in altitude than the Štrbský threshold). An insufficient communication infrastructure and consequent underdevelopment (even during the existence of Czechoslovakia) may have been partially caused by the fact that the territory is settled by a Hungarian minority (Halás 2005: 259–260). It is a very delicate and problematic issue and this fact has not been sufficiently taken into account in historical geographical and human geographical literature.

It is interesting that there was no effort to develop a southern passage during 40 years of socialism. In this period, the strong levelling state policy minimised all occurring disparities. At present, it can be assumed that the policy diminished vertical (between social strata) rather than horizontal (between regions) disparities. Moreover, the economically weaker regions were stimulated more by a financial redistribution than by support for sophisticated economic activities and refurbishment of communication infrastructure. The result is that during 25 years of economic transition regional disparities have radically deepened and the South Slovakian basin (together with some parts of Eastern Slovakia) has become a distinct periphery.

Regional disparities in Slovakia can be documented very well through the identification of peripheral regions (Halás 2008). Apart from their delimitation, this work also identified types of peripheries according to their worsened results in a number of indexes, grouped according to factors such as: human resources (including the educational structure, age structure and migration balance), economic potential (including the unemployment rate, rate of entrepreneurial activity and share of employment in so-called progressive services), personal equipment (including the ownership of a car, mobile phone and equipment of the household with a washing machine and computer), accessibility of centres (including the distance from the regional capital and from town with the population larger than 20,000), and cumulated peripherality (see Fig. 1). For each group, a municipality that occurred in the lower quintile was considered to be a peripheral municipality. Four groups of peripheral municipalities have been identified in this way, while the municipalities occurring in the lower quintile in three or four groups have been subsumed into the group denoted as the cumulated peripherality.

Spatial identification of peripheries also shows a line documented elsewhere several times, which divides Slovakia into the "rich north-west" and the "poor south-east" (Fig. 1). This line is relatively distinct and it can be found in almost all studies on regional disparities in Slovakia (Korec 2005; Džupinová et al. 2008; Rosina and Hurbánek 2013). The areas to the south of the line are between regions that are, in the long-term, lagging behind the most economically, with a high unemployment rate, low social capital, and insufficient social and technical infrastructure. Regional centres in these areas are smaller than in the north-western part of Slovakia. One of the hypotheses of this paper is that the spatial influence of these centres (e.g.

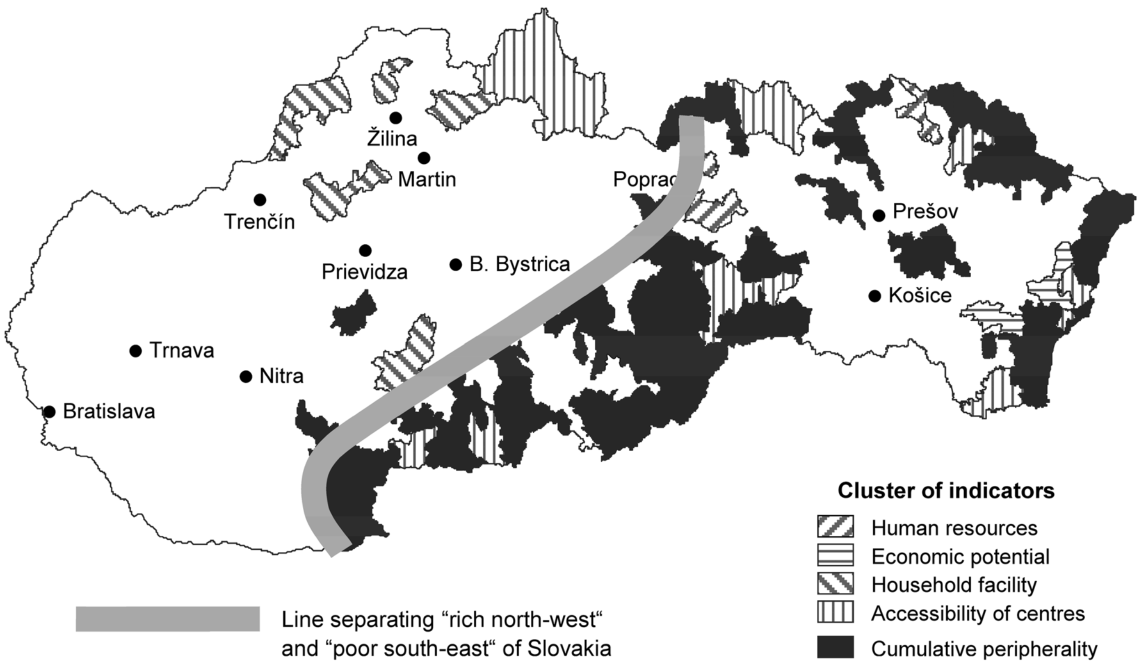

Fig. 1 Regional disparities and location of peripheral regions of Slovakia; according to Halás (2008)

Lučenec, Rimavská Sobota and Rožňava) is comparable to the spatial influence of larger centres in north-west Slovakia.

2.2 Spatial interaction modelling and distance-decay function

The issue of distance-decay models is a multi-disciplinary research theme that is pursued by both natural and social scientific disciplines. It is a standard theme for environmental science and for research into the laws of spatial distribution of flora, fauna and their elements. Increasing distance causes, for instance, a decrease in biological similarity of organisms and a decrease in interaction intensity between a wide spectrum of organisms and ecosystems (Soininen et al. 2007). Distance-decay models analysing links and interactions can be found in recent natural scientific studies (e.g. Sileshi and Arshad 2012; Morlon et al. 2008), but they also have a long-term research tradition. Some compound distance-decay functions were named after their authors, such as the Richards' function (Richards 1959) and the Box-Cox function (Box and Cox 1964). The use of relevant distance-decay functions for organisms and ecosystems is important, but regarding infrastructure planning in urban environments and their respective functional urban regions, it is the actual daily movement of the population that is crucial.

The first efforts to capture the changes in spatial interactions that occur with increasing distance were made in the social sciences in the last quarter of the Nineteenth century. Ravenstein (1885), inspired by physical relations, tried to express the intensity of migration flows between British regions. His thoughts were further developed particularly after the Second World War. Stewart (1948) ascertained an analogy between the spatial behaviour of individuals and the movements of molecules. Based on these findings, he expressed the concept of so-called social physics and he defined the terms demographic force and demographic potential. In fact, it was the analogy of Newton's law of gravitation to social and economic disciplines, when he defined the demographic force between cities as the quotient of the product of populations of both cities and the square root of their distance. Thus, he de facto introduced the distance-decay function, which was given by an inverted value of squared distance. In the economic environment a similar concept was used in the inter-war period by Reilly (1929, 1931), who defined the law of retail gravitation on the basis of real interactions observed in Texas in the 1920s. This basic thought was developed and used for retail planning in the following decades. Reilly's law was extended by Converse (1949: 379), who expressed mathematically and precisely the breaking point and boundary line between the spheres of influence of two shopping malls. Huff (1964: 36–37) expressed the theoretical probability for choice of individual shopping malls by a customer. Thompson (1966) hints at further possibilities for the use of Reilly's Law in both general and specific spatial themes (Reilly's Law has been used until now, e.g. Řehák et al. 2009; Kraft and Blažek 2012).

Taylor (1971) published his pioneering work that pursued the distance-decay function in detail. He drew attention to the fact that not all phenomena behave similarly to the physical analogy in social and economic environments. He puts forward several alternatives and types of distance-decay functions: power function, exponential function, logarithmic function and the so-called Pareto function (Taylor 1971: 229–230). These types were identified based on actual spatial flows or interactions and they were expressed graphically, often with a logarithmic transformation of the input data. Many of these functions have been used until now to model spatial interactions, for instance Pareto function and its modifications were used by Aubigny et al. (2000), Grasland and Potrykowska (2002), Klapka et al. (2013), and Liu et al. (2014), in an unpublished official working paper on the modelling of decay in spatial interactions (Grasland 1996). However, the suitability of these basic functions for some tasks was questioned, for instance by Fotheringham (1981), and De Vries et al. (2009).

Despite these facts the above-mentioned functions remain as the basic distance-decay functions for spatial interaction modelling. Recently, they have been modified as compound distance-decay functions, which often have an inflexion point and better capture changes in interaction intensity that depend on distance. These functions frequently have two parameters. There can be even more parameters with regard to how and with what precision the interaction changes should be captured. Several studies which put forth new possibilities for studying the spatial influence of regional centres were concerned with the changes in intensity of travel-to-work flows. Instead of distance Johansson et al. (2002) use the time accessibility of a centre and also the intra-urban flows. The research presented in this paper only uses flows between basic spatial units (municipalities), since the intra-unit flows are not available. A complex compound function was used by Ubøe (2004), who used aggregated travel-to-work flows in the modelling. O'Kelly and Niedzielski (2009) discuss the values for parameters and their influence on the course and shape of the function with regard to the size of in-commuting centres. Distance-decay functions are frequently used for modelling and planning of the transport flows (e.g. Timmermans et al. 2003; Martínez and Viegas 2013).

Martínez and Viegas (2013) do not use statistical data in their work, but use the results of their research on the subjective perception of the terms "near" and "far". Similar to Johansson et al. (2002), they use time accessibility instead of distance. This approach is relevant in cases where only one travel mode is analysed. If it is not possible to identify the travel mode (individual vehicle, mass transportation, modal shift) it is necessary to employ the Euclidean distance (also used in this paper). Apart from basic functions (power, exponential), the work of Martínez and Viegas (2013) proposed an overview of more complex functions, such as Tanner function, Box-Cox function and Richards' function. These functions are considered to be more flexible (they were theoretically described and applied by Ortúzar and Willumsen 2011; Tiefelsdorf 2003 and Willigers and Floor 2007). The most complex, Richards' function, has four parameters and is able to capture the changes in interaction in detail. Its disadvantage is that it can be used only for individual instances (of regional centres) and cannot be generalised. The basic notation of functions that are most frequently used in spatial interaction modelling is given in Table 1. The compound power exponential function was used by Halás et al. (2014) in a study of travel-to-work flows to regional centres of the Czech Republic. This function approximates very well the decrease in interaction intensities with increasing distance from a centre. Therefore, it will be used in this paper together with the Richards' function and their course will be compared. Some of the reasons and arguments for their application will be detailed in the empirical sections of the paper.

A specific instance of the distance-decay function is represented by functions of two variables: apart from the distance, the selected size characteristic of a centre is used. Mozolin et al. (2000: 66–68) modelled flows in parts of the Atlanta metropolitan region in this way. The number of employed persons was used as the other variable and the results can be expressed graphically in the 3D model (a similar 3D model is used in this paper as well). The spatial interaction modelling uses as an analogy (similar to the Richards' function) a knowledge of biology and medicine, e.g. neural network modelling as a template for the approximation of transport and travel-to-work flows (Dougherty 1995; Gopal and Fischer 1996; Mozolin 1997; Himanen et al. 1998). More recently, the issue of changes in interactions with increasing distance is discussed in relation to the transport situation in an intra-urban

Table 1 Basic distance-decay functions used in spatial interaction modelling

Function	Basic form
Power	$d^{-\alpha}$
Pareto	$(1+d)^{-\alpha}$
Exponential	$\exp(-\alpha \cdot d)$
Tanner	$d^{-\alpha} \cdot \exp(-\beta \cdot d)$
Power-exponential	$\exp(-\alpha \cdot d^{\beta})$
Box-Cox	$\begin{cases} \exp\left(-\alpha \frac{d^{\gamma}-1}{\gamma}\right); \gamma \neq 0 \\ d^{-\alpha}; \gamma = 0 \end{cases}$
Richards	$1 - \frac{1}{[1+\alpha \cdot \exp(-\beta \cdot d + \gamma)]^{1/\delta}}$

environment (Gutiérrez et al. 2011; Roth et al. 2011; Mamuna et al. 2013), as well as the connection between distance and the willingness of job seekers to commute (Heldt Cassel et al. 2013), their detailed socio-economic characteristics (Cheng and Bertolini 2013), and with the preferences of the population in their choice of place of residence with regard to job location (Ibeas et al. 2013).

3 Method

Data on migration, particularly on labour and school commuting, is the basic information on the spatial mobility of a population and on spatial interactions. The paper builds on the labour commuting data (daily travel-to-work flows) from the 2001 census. Newer data from 2011 are not yet at our disposal and their information value will be significantly lower according to the statistical office. The question on labour commuting was not compulsory in 2011 and a high percentage of the population did not provide the necessary information. The distance for daily movements is limited and the distance-decay function simply has to reach 0 or it has to approach 0 (i.e. the x axis) asymptotically.

The distance-decay function for a particular centre has been constructed in the following way: the x axis shows the distances (in km) from various municipalities to the given regional centre, while the y axis gives the portion of daily commuters from these municipalities to the given centre out of the total number of local daily out-commuters. These values occur in the interval between 0–1 (or 0–100 %) and are referred to as the interaction intensity. The interaction intensity in the centre is not measurable of course; we posit, then, that the value of the interaction intensity in the centre, i.e. at the zero distance, is 1 or 100 %. Each municipality is represented by a single point on a graph. In relation to the location of these points (municipalities) on the graph coordinates, the optimal distance-decay function for the particular centre is approximated. The graph values are not transformed, since in this form the information capability of the distance-decay function is greatest (e.g. for further three-dimensional expressions, or for other applications). The model uses road distances, given to the nearest 0.1 km, which have been provided as the fastest variant by the route planner mapy.cz (http://www.mapy.cz). The Euclidean distance, not time distance, has been used, since the available data provide only sums of commuting flows and each municipality favours a different modal share for labour commuting according to the specific geographical conditions. The use of the time distance is not possible in this case regarding the method's correctness.

The data have been processed for regional centres with more than 40 thousand inhabitants (15 centres). This criterion for the selection of potential centres at the regional level has been derived from the population of the smallest current administrative capital (Trenčín, exceeding 50,000 inhabitants). Thus, the level of 40,000 inhabitants is able to capture all potential regional centres with a provision. Three other centres have also been included in the analysis, those that, in some versions of the proposals for administrative division of Slovakia, had appeared as regional capitals. They are located in areas with lower population density and economic potential and thus their spatial influence is assumed to be spatially larger (Komárno, Lučenec and Rimavská Sobota). The choice of centres has not been random, the spatial influence of the centres at the regional level (i.e. approximately, NUTS 3 regions) has been analysed by using Reilly's Model, for instance by Halás and Klapka (2012).

The first tests have applied standard functions with no inflection point, which was previously the most frequently used in the literature for interaction approximation (such as power, exponential, Pareto functions etc.). However, these functions do not approximate the daily movements of population very well, they either go beyond the value 1 for short distances from the centre, or else they run totally outside the main point cluster.

According to the distribution of points on a graph, it is quite clear that an application of the bell-shaped function, a decreasing function with an inflection point, changing its curve from concave to convex and beginning at the point [0; 1], or at least [0; x]: $0 < x \leq 1$, will be more favourable. To express such a function, two factors are crucial: the extent of the regional centre's influence and the manner in which this influence decreases. Therefore, at least two variable parameters have to be employed to express the optimal distance-decay function for a regional centre. Two relevant functions have been tested. The simplest function that obeys these conditions is a compound power-exponential function in the form:

$$f(d) = \exp(-\alpha \cdot \mathrm{d}^{\beta}),$$

where d is the distance from the centre; α, β are parameters, $\alpha > 0$, $\beta > 0$. This function has been already used for similar analysis in the example of the regional centres of the Czech Republic. It belongs to the group of more complex functions that better approximate the decrease in the intensity of daily travel-to-work flows, and that are more flexible with regard to the shape of the curve.

A similarly shaped distance-decay function was presented by Martínez and Viegas (2013), while in its most complicated form this function had four parameters. The advantage of the function proposed in this paper is that it sufficiently approximates the intensity decrease of the daily travel-to-work flows using a simpler form and only two parameters. Unlike the afore-mentioned work (Martínez and Viegas 2013), the x axis represents the Euclidean distance, not the time distance, since the daily travel-to-work flows are produced by different transport types and the conversion of the Euclidean distance to time distance would require a weighting of the transport types, and thus too many subjective inputs would enter the function. In the case of $0 < \beta \leq 1$ the function does not have a bell shape but it resembles a simple exponential function (if $\beta = 1$ it is the simple exponential function). The second tested function is the Richards' function, which is a part of the logistic function family $\left(\frac{1}{1+\exp(-\beta x)}\right)$. Its notation is as follows:

$$1 - \frac{1}{\left[1 + \alpha \cdot \exp(-\beta \cdot d + \gamma)\right]^{1/\delta}},$$

where d is the distance from the centre and α, β, γ, δ are the parameters $\alpha > 0$, $\beta > 0$, $\delta > 0$. As this function has up to four parameters, it approximates very well the interaction intensity decrease. It was used for the first time by Richards (1959) in the field of botany to reproduce empirical data on the growth of plants. Recently, it was used by Martínez and Viegas (2013) for the modelling of transport flows.

The important indicator which expresses the extent of the influence of regional centres is the area S below the curve. In a standard case, it could be expressed by a certain integral:

$$S = \int_0^\infty f(d)dd,$$

but if both selected functions are transcendental, there is an anti-derivative function counterpart, which cannot be expressed by an elementary function of a final shape. Therefore, we have applied the approximate rectangular method with a calculation step of 10^{-5} km (i.e., 1 cm). Specifically, S expresses the theoretical distance of the 100 % extent of a regional centre (Fig. 2), hereafter called the "radius of influence" (because we employ road distances the potential shape of the area of influence of a regional centre will not necessarily be an exact circle). The S value sets the radius of influence and the area of influence of the centre but it does not express the population size of the area.

The parameters α, β in the power-exponential function are mutually dependent, while individually they are minimally dependent on the size of a centre, with more significant dependency on the size of a centre identifiable only when they are combined. Regarding the mutual dependency of the parameters α, β, just one of them is sufficient to assess the resulting typology or classification of the distance-decay functions for individual centres. The β parameter has been selected, since it better controls the shape of the function (the measure of the "bell bent") including the position of an inflexion point. The change in the shape of functions is determined by the β parameter in the resulting expression on the x–y graph (Fig. 8). The spatial extent of regional centres is assessed by the most representative parameter, which is the radius of influence S in this case. Figure 8 graphically expresses the dependency between the spatial influence of individual centres (x axis) and the shape of the curve for the distance-decay function (the y axis); however, no significant clusters (i.e. types) of the regional centres have been formed. Therefore, the resulting typology is based on the value of the spatial influence of individual centres in relation to the population size of centres (Fig. 12). This dependency is also used for the construction of a universal distance-decay function (i.e. the dependency of the interaction intensity not only on the distance but also on the population of the regional centre).

The method described so far is virtually analogical to the method published by Halás et al. (2014), even though the Richards' function has also been tested for individual functions. The purpose is to assess the utility of the same function for the analysis of the same index in various territories. It will also enable a better generalisation of acquired knowledge. In the next step, the procedure is original. The dependency of the size of a regional centre on the radius of influence is assessed. This dependency is not assessed directly to reach better graphical expression, but it is mediated by the dependency of the population and the α_2 parameter. Further methodological procedures for the construction of the universal distance-decay function and typology are directly connected with the results that show distance-decay function shapes for particular centres, therefore the method will be described, together with a presentation of the results in the following paragraphs.

4 Results

4.1 Basic (individual) distance-decay function

The resulting shape of the distance-decay functions for regional centres captures well the regional and settlement structure of Slovakia. The dominance of Bratislava and its regular coexistence with Košice as the secondary centre dominant in the east of Slovakia and with some other regional capitals such as Prešov, Banská Bystrica, Žilina, Trenčín and Nitra, is evident. The dominance of Bratislava and Košice is confirmed by the fact that the values of the radius of influence show that the spatial influence of Bratislava is 2.8 times larger and that of Košice is twice the spatial influence of Prešov, which ranks third (for spatial extent of the radius of influence see Fig. 3).

The extremely large radius of influence of Bratislava actually represents a slightly smaller area because of the

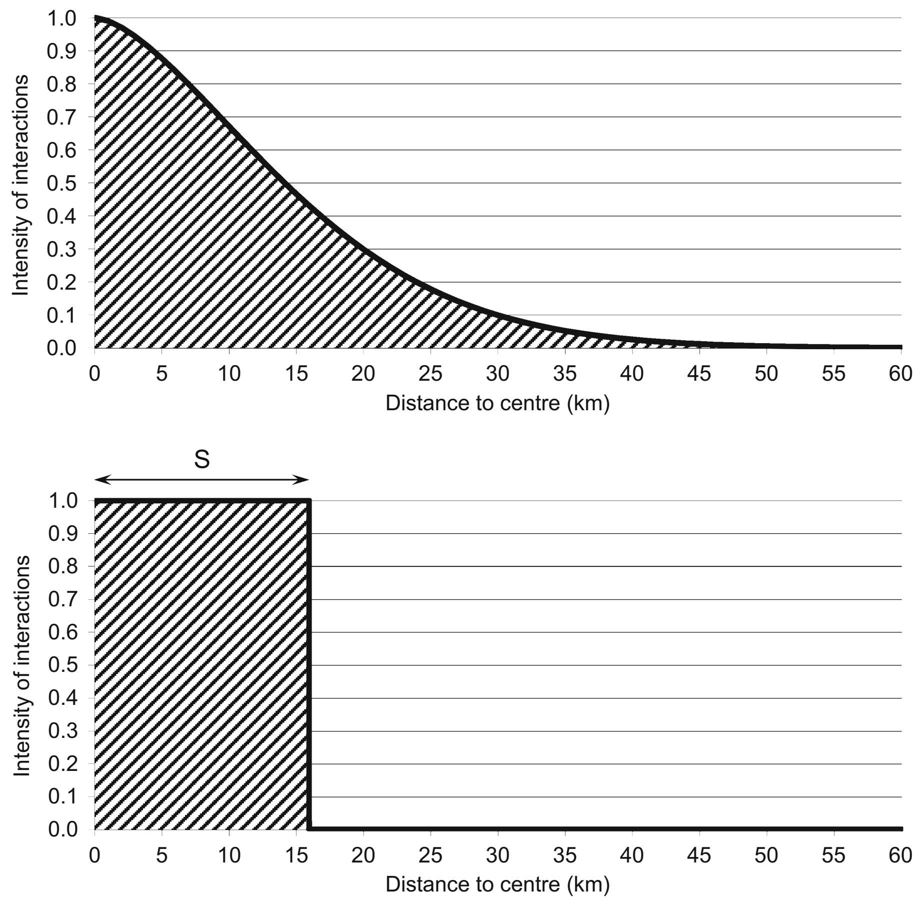

Fig. 2 Graphical expression of the radius of influence *S* for regional centres

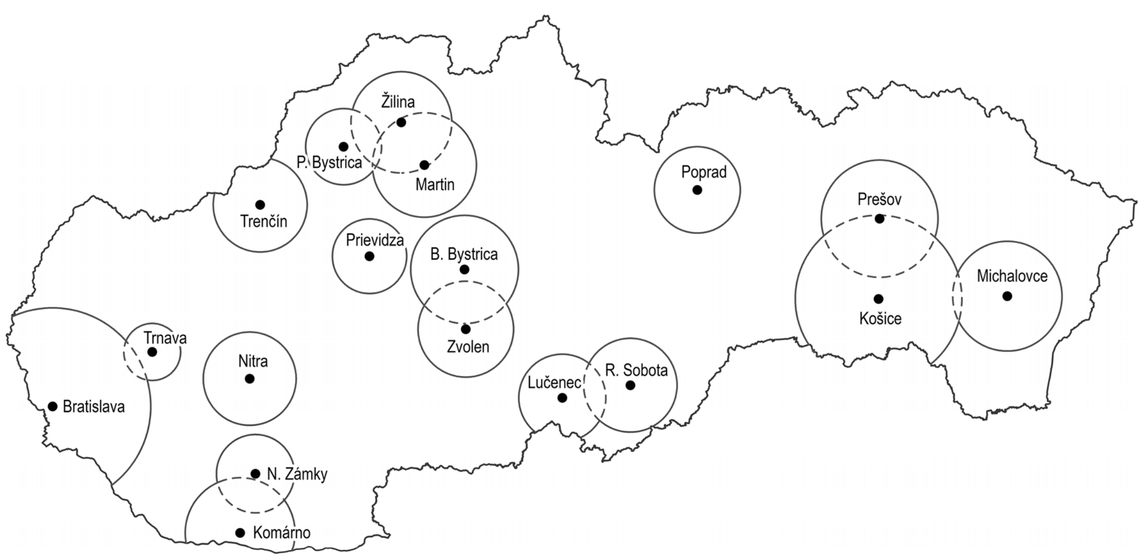

Fig. 3 Spatial extent of the radius of influence of regional centres

location of the capital in the vicinity of the state border with Austria and Hungary (only Slovak municipalities in the hinterlands of regional centres entered the analysis. However, recently the influence of Bratislava has also spread to Hungary and Austria). The real spatial influence does not conform to the radius of influence, particularly in the case of Komárno that lies just on the border. Its real spatial influence is actually half as large (in simple terms instead of a circle it is a semi-circle). The remaining centres that have been analysed are not located on the border. The distance-decay function generally aptly captures the daily movements of population to regional centres. This is supported by the high values of the coefficient of determination. The high information value of the distance-decay function has been asserted for all analysed centres. The value of the coefficient of determination increases with the homogeneity of geographical environment, both natural (in areas without prominent morphological barriers) and socio-economic (in areas with even spatial distribution of settlements).

The values of the resulting parameters for individual distance-decay functions for all analysed centres are presented in Table 2. Apart from the values of basic parameters, the coefficient of determination and the radius of influence *S*, the value of the mean distance is also shown, which is the average distance of labour commuting for a particular centre. This value corresponds, with a few minor exceptions, to the radius of influence. It is significantly different in the case of Trnava, which has a very small radius of influence and a long mean distance. This is caused by the relatively high travel-to-work flow from Bratislava to Trnava that significantly increases the value of the mean distance. However, the value of the radius of influence is not affected, since Bratislava is only one of many points on the graph of the distance-decay function for Trnava, and moreover, in a relativised expression this particular flow has a standard value.

Individual distance-decay functions for the three largest centres are presented in Figs. 4, 5 and 6, for regional capitals in Fig. 7. On the basis of these figures and data from Table 2, it is possible to formulate a comparison of the applicability of the power exponential function and the Richards' functions for the approximation of daily travel-to-work flows:

- The power exponential function starts at the point [0; 1], while the Richards' function does not respect the theoretical postulate that the interaction intensity in a centre is 1. It could be amended through the function notation, but the coefficient of determination would decrease.
- The radius of influence *S* for both functions significantly correlates with the population size of a centre (linear dependency), while power exponential functions manifest slightly higher correlation coefficients.
- Both functions aptly approximate daily travel-to-work flows. The coefficient of determination shows comparable values, in the case of the Richards' function it is slightly higher (Table 2).
- The power exponential function has simpler and more transparent notation, which provides a better opportunity for generalisations and the formulation of a universal distance-decay function.

Table 2 Parameters for distance-decay functions for daily travel-to-work to regional centres

Centre	Population	Mean distance	Richards function						Power-exponential function			
			α	β	γ	δ	R^2	S	A	β	R^2	S
Bratislava	425,533	51.6	0.0405	0.0453	19.8386	0.0056	82.5	36.9	0.0049	1.4331	82.4	37.0
Košice	235,281	31.7	0.6658	0.1810	−41.8643	2.3604	89.3	30.2	0.0001	2.5017	88.7	31.4
Prešov	92,380	19.4	2.4035	0.2610	−26.8238	3.2418	86.5	20.5	0.0009	2.1995	85.0	22.0
Komárno	36,667	21.8	1.8398	0.1447	−19.5209	1.5059	77.2	19.6	0.0032	1.8202	77.0	20.7
Michalovce	40,721	16.6	0.0656	0.0790	8.8731	0.0121	64.7	20.0	0.0177	1.3003	64.6	20.6
Banská Bystrica	81,961	24.8	0.4733	0.1042	−13.5036	0.3583	83.6	20.1	0.0059	1.6413	83.7	20.3
Martin	58,814	13.4	2.1042	0.2036	−26.7385	3.8992	89.2	16.9	0.0249	1.2157	85.5	19.6
Žilina	85,278	17.9	0.6613	0.1345	−19.5316	0.8801	84.4	18.5	0.0055	1.7015	84.2	19.0
Zvolen	43,488	20.4	0.0425	0.0813	8.5535	0.0095	61.6	17.7	0.0304	1.1842	61.5	18.0
Trenčín	57,968	18.2	0.8648	0.1639	−21.4949	1.5207	78.6	16.9	0.0069	1.6676	78.3	17.7
Rimavská Sobota	24,446	16.4	0.0428	0.1032	3.7542	0.0085	72.4	17.6	0.0110	1.5172	72.3	17.6
Nitra	86,580	21.0	0.8753	0.2061	−20.7630	1.5309	84.4	16.8	0.0019	2.1076	84.0	17.5
Lučenec	28,491	14.0	0.0687	0.1265	3.4917	0.0096	76.0	16.6	0.0055	1.7858	75.7	16.4
Poprad	55,680	16.4	0.0236	0.0728	0.3896	0.0155	65.7	14.7	0.0928	0.8738	65.4	16.2
Nové Zámky	41,669	16.2	0.7583	0.1146	−11.7165	0.8183	64.7	13.6	0.0382	1.1890	64.6	14.7
Považská Bystrica	42,514	15.1	3.7464	0.5953	−20.3941	7.1124	73.6	12.8	0.0043	1.9592	71.9	14.3
Prievidza	53,418	13.7	1.4226	0.1533	−17.8466	2.4914	83.1	11.9	0.0713	1.0010	80.4	14.0
Trnava	69,488	28.5	0.2969	0.1152	23.3616	0.0123	78.3	10.0	0.0913	1.0099	78.3	10.7

α, β, γ, $\hat{o}$ parameters, R^2 coefficient of determination, S radius of influence

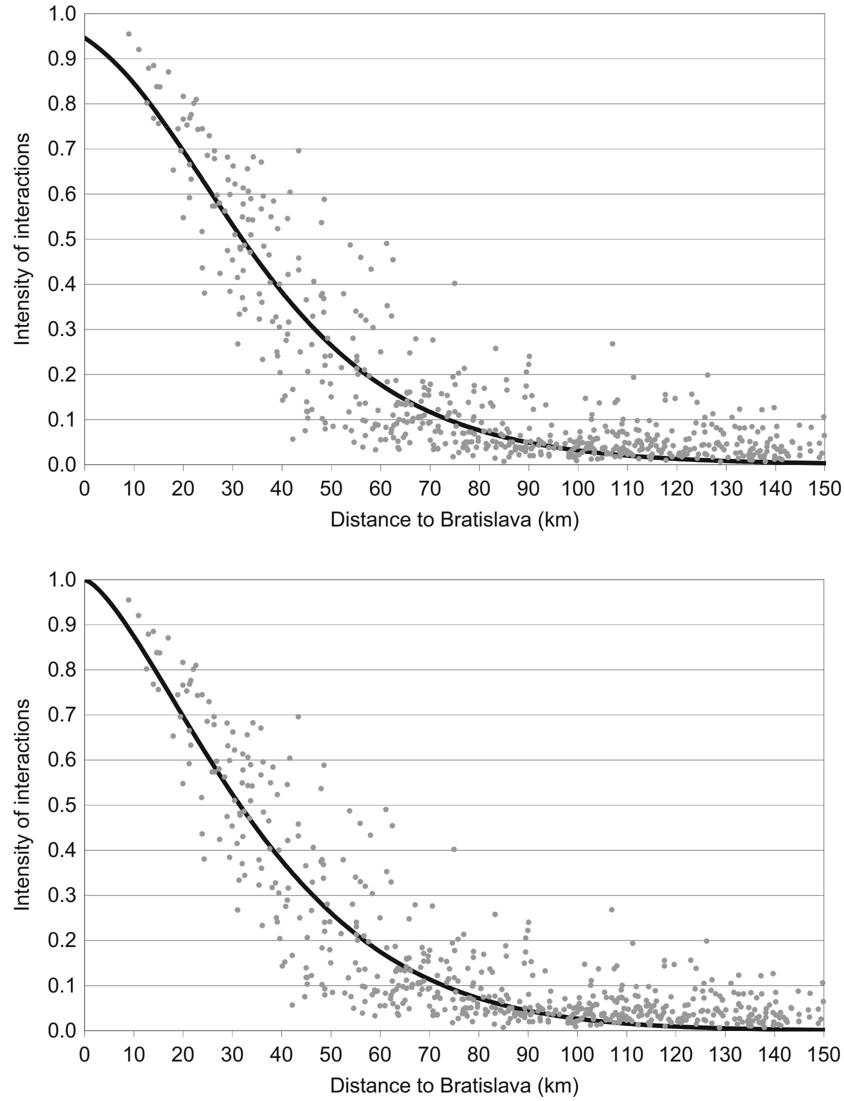

Fig. 4 Distance-decay function for daily travel-to-work to Bratislava (Richards function and power-exponential function)

This assessment suggests that both the power exponential function and the Richards' function can be used to construct distance-decay functions for daily travel-to-work flows, but the former appears to be more suitable for generalisations and further use. Therefore, the power exponential function will be used later in this paper.

The relation between the spatial influence of regional centres (i.e. the radius of influence *S*) and the shape of individual distance-decay functions (i.e. the β parameter) is presented in Fig. 8. The position of individual centres in the graph determines the character of distance-decay functions for the analysed centres, which is detailed in the preceding paragraphs. Provided that regional centres do not form significant clusters (types), the resulting typology of centres according to the distance-decay functions is developed in another way (see the section Typology of regional centres according to the distance-decay functions).

The values of the radius of influence *S* confirm the relevance and representative nature of the sample of analysed centres. Provided the radius of influence is a circle of 100 % of the influence of a centre (statistically), the calculation of the area for such a circle gives an acreage of 100 % of the influence of a centre (again statistically). If areas of 100 % influence out of 18 analysed centres are totalled, the acreage of 23,140 km^2 is gained, which is 47.2 % of the total area of Slovakia. The analysed centres have a spatial influence over almost half of the area of Slovakia after statistical calculations, while the remaining 52.8 % of the area comes under the influence of other (smaller) regional centres.

The value of the β parameter varies between 1 and 2 in most cases, the extreme case being Košice reaching 2.50. A minimal value for the β parameter is reached in the case of Poprad (0.87). The graphical comparison of high and

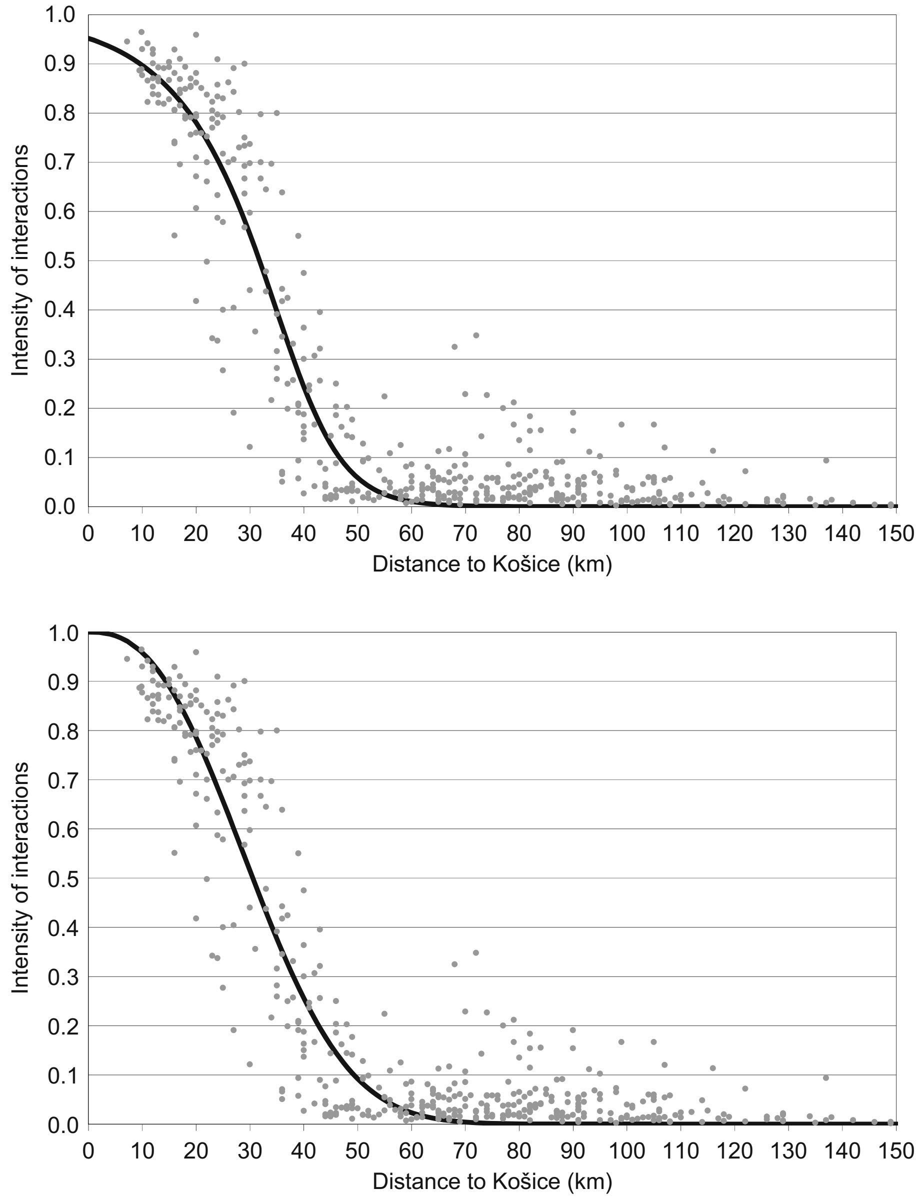

Fig. 5 Distance-decay function for daily travel-to-work to Košice (Richards function and power-exponential function)

low value of the β parameter is shown in the example of Nitra and Poprad, which both have similar radii of influence *S*. While Nitra with a high value for the β parameter has strong interactions in its immediate hinterland (most out-commuters from the immediate hinterland travel to Nitra to work), the interaction intensities decrease sharply with an increasing distance. This can be documented on the 3D model, where the shape of the distance-decay function for Nitra differs from the distance-decay function for Poprad, which has a low value for the β parameter (Fig. 9). In the case of Poprad, the interaction intensity decreases immediately behind the centre frontier, however, certain values of the interaction intensity still occur in the longer distance.

4.2 Universal distance-decay function

The preceding results have determined individual (specific) distance-decay functions for each regional centre separately. Regarding the fact that the variability of the β parameter for the optimal power exponential function is low, a universal distance-decay function can be estimated, provided that a constant value is set for this parameter. The constant value for the β parameter has been estimated by the weighted mean of the β parameters for all analysed centres, while the weight is set by the values for the coefficient of determination R^2. The resulting unified β parameter denoted as β_2 has a value 1.59 (the value is represented by the line in Fig. 8). Optimal values of the α_2

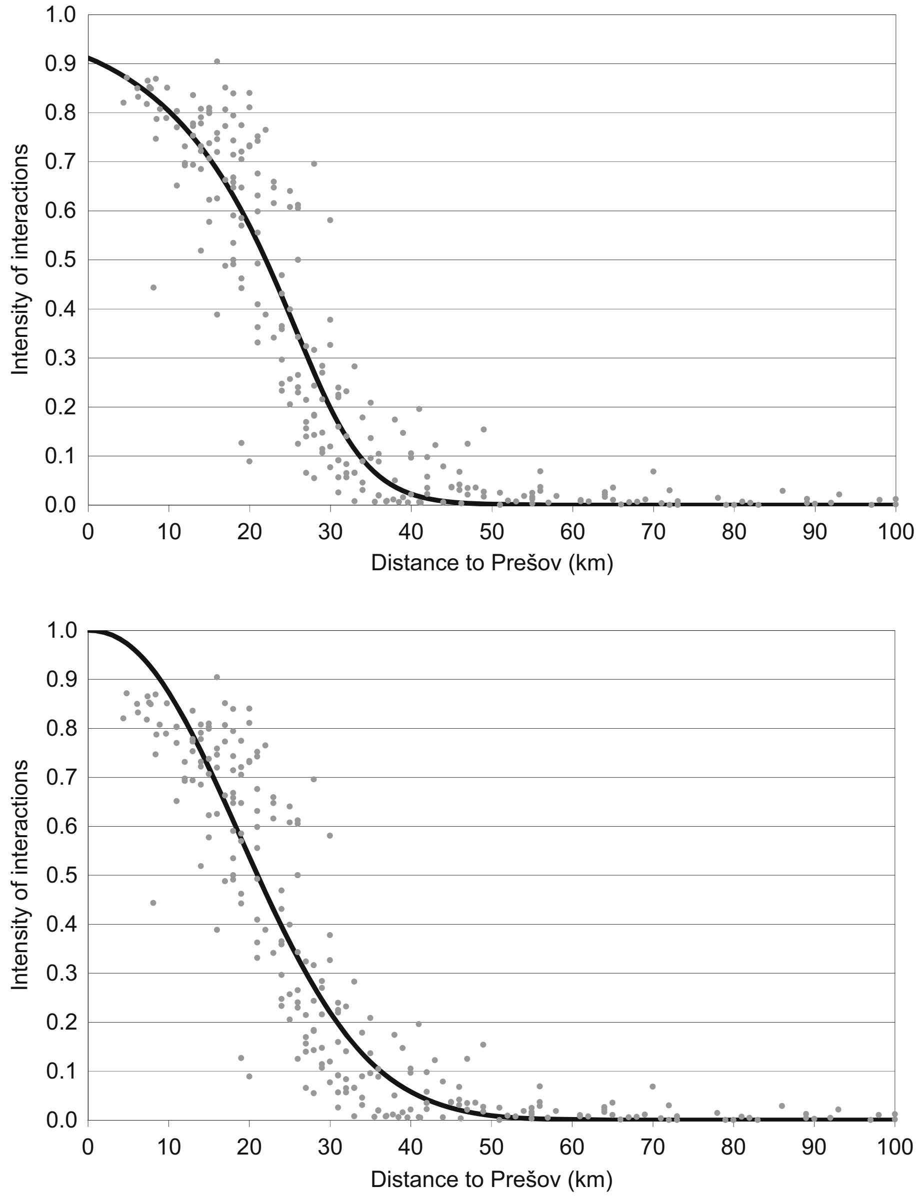

Fig. 6 Distance-decay function for daily travel-to-work to Prešov (Richards function and power-exponential function)

parameter for $\beta_2 = 1.59$ are shown in Table 3. The values of the coefficients of determination for individual centres prove that such distance-decay functions do not significantly differ in their information value (Table 2) from the optimal individual distance-decay functions, and that the construction of the universal distance-decay function has its reason.

The constant β parameter and estimation of optimal value of the α_2 parameter for each regional centre provide values for the α_2 parameter, which are significantly dependent on the size characteristics of regional centres. This dependency can be determined by the regression model. A suitable transformation (logarithmic) of the α_2 parameter, actually of the population of regional centres, to the linear regression model provides a dependency expressed by the function noted in Fig. 10. The linear dependency between the transformed α_2 parameter and the transformed population gives a correlation coefficient of the value 0.85. The same procedure has been applied to express the dependency of the α_2 parameter on the job positions. Even though the job positions could appear as more suitable data for an expression of the spatial influence of a centre based on the travel-to-work flows, the correlation coefficient between transformed α_2 parameters and transformed job positions is slightly lower (0.84). It is given by the strong social polarisation of Slovakia. In poorer regions, the centres have a relatively large spatial influence but a smaller population, while high unemployment rate significantly decreases the number of job positions and thus the results are partially distorted (two regional centres that

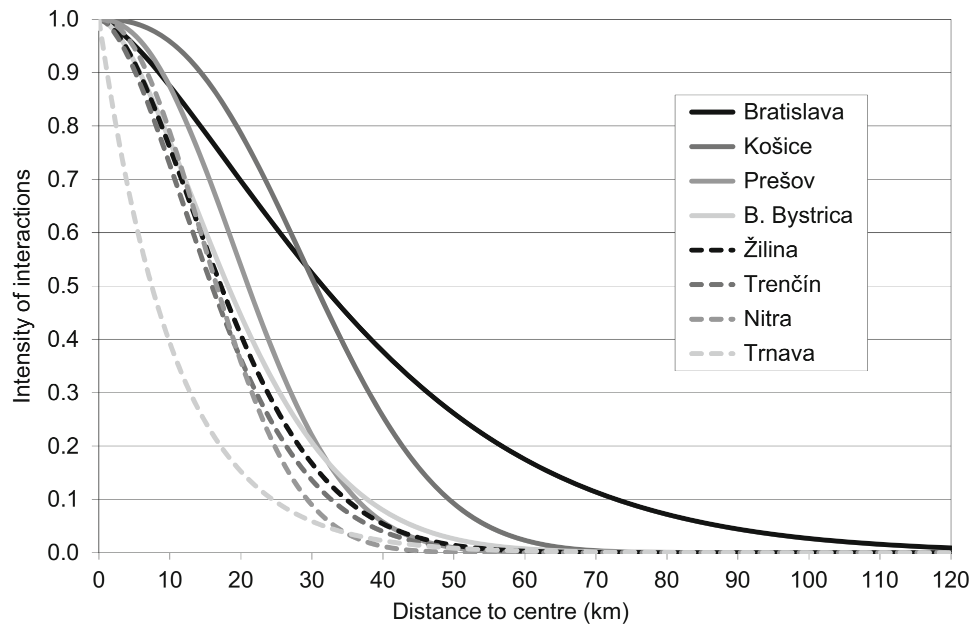

Fig. 7 Distance-decay function for daily travel-to-work to regional capitals (power-exponential)

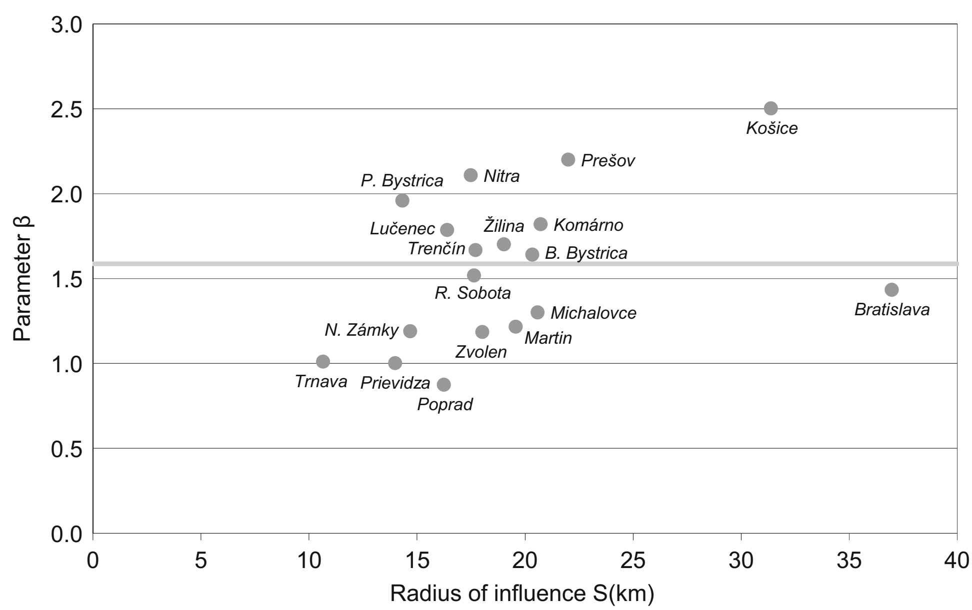

Fig. 8 Relation between the radius of influence S and β parameter for the distance-decay functions for regional centres

could distort the results have been left out of the regression model: Komárno lies on the state border and its radius of influence S does not correspond to its spatial influence, Trnava is located in the hinterland of Bratislava and its radius of influence S is therefore considerably lowered).

A more accurate approximation and results would be gained in cases where the individual analyses were carried out for core and peripheral areas of Slovakia. On the basis of the already-mentioned regional studies the territory of Slovakia could be divided into the "rich north-west" and the "poor south-east" (e.g. Džupinová et al. 2008; Halás 2008; Korec 2005). An imaginary dividing line connects Levice to Poprad in its generalised form (Fig. 1). From the centres entering this analysis Banská Bystrica, Bratislava, Martin, Nitra, Nové Zámky, Poprad, Považská Bystrica, Prievidza, Trenčín, Zvolen and Žilina belong to the core area (the rich north-west), and Košice, Lučenec, Michalovce, Prešov and Rimavská Sobota belong to the peripheral area (the poor south-east). In the core area, the value of the correlation coefficient between transformed α_2 parameter and transformed population is 0.94 for the expression of linear dependency, in the peripheral area this number is 0.98.

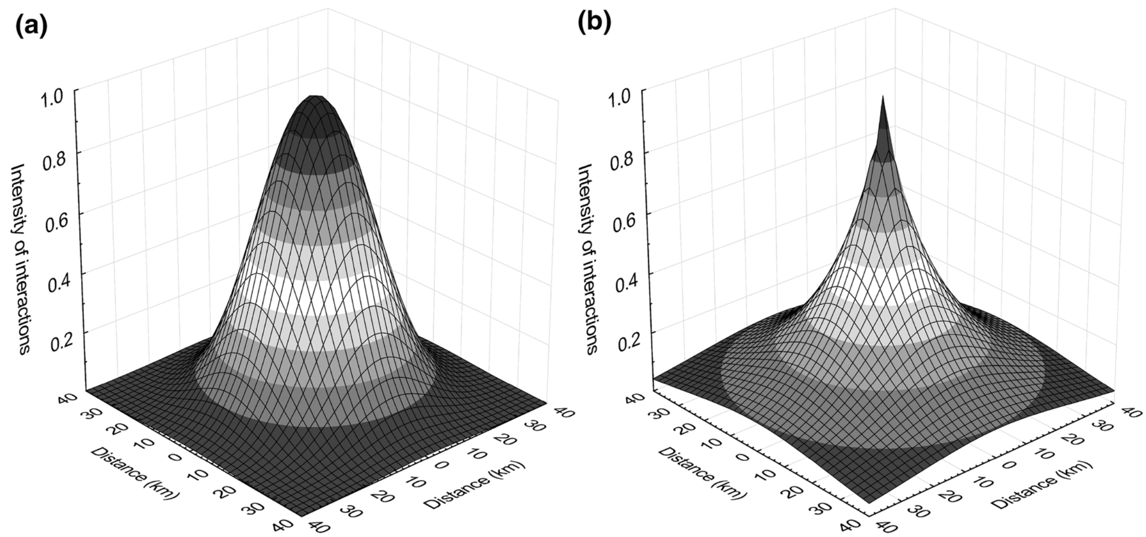

Fig. 9 3D model for distance-decay functions for daily travel-to-work flows to Nitra (**a**) and Poprad (**b**)

Table 3 Parameters for distance-decay functions for daily travel-to-work flows to regional centres (power-exponential function for the constant parameter $\beta_2 = 1.59$)

Centre	α_2	β_2	R^2
Bratislava	0.0027	1.59	81.6
Košice	0.0034	1.59	86.0
Prešov	0.0060	1.59	83.0
Komárno	0.0068	1.59	76.4
Michalovce	0.0073	1.59	63.1
Banská Bystrica	0.0070	1.59	83.6
Martin	0.0082	1.59	83.4
Žilina	0.0077	1.59	84.1
Zvolen	0.0090	1.59	59.4
Trenčín	0.0087	1.59	78.2
Rimavská Sobota	0.0089	1.59	72.2
Nitra	0.0090	1.59	82.5
Lučenec	0.0096	1.59	75.3
Poprad	0.0120	1.59	50.7
Nové Zámky	0.0117	1.59	61.5
Považská Bystrica	0.0120	1.59	70.8
Prievidza	0.0127	1.59	73.1
Trnava	0.0186	1.59	71.3

On the basis of the dependency between transformed α_2 parameters and transformed population confirmed by the linear regression model, the distance-decay function for regional centres can be universally estimated, depending on the population of regional centres, which is the most general and universal quantitative characteristic of regional centres. An optimal function for an expression of changes in interaction intensities dependent on the distance from a regional centre and the population is:

$$f(d, p) = \exp(-2.1621 \cdot p^{-0.5061} \cdot d^{1.59}),$$

where p is the population of a regional centre. It is a function of two variables: distance from the regional centre d and population size of the regional centre p. A graph for this function is given in Fig. 11. Calculation of that relationship is as follows:

$$y = 0.5061 \cdot x - 0.7711; y = -\ln(\alpha_2); x = \ln(p);$$
$$-\ln(\alpha_2) = 0.5061 \cdot \ln(p) - 0.7711$$
$$\Rightarrow \alpha_2 = \exp(-0.5061 \cdot \ln(p) + 0.7711)$$
$$\Rightarrow \alpha_2 = 2.1621 \cdot p^{-0.5061};$$
$$f(d) = \exp(-\alpha_2 \cdot d^{\beta_2})$$
$$\Rightarrow f(d, p) = \exp(-2.1621 \cdot p^{-0.5061} \cdot d^{1.59}).$$

4.3 Typology of regional centres according to the distance-decay functions

The typology of distance-decay functions draws on the relation between the spatial influence of a regional centre and its population size. The spatial influence can be represented either by the already mentioned radius of influence S, or by the α_2 parameter. The analyses provide almost identical results in both cases. Regarding the analysis of the α_2 parameter dependency on the population which was carried out in the preceding section, the results will be also used for the typology of regional centres according to the distance-decay functions.

Trnava is a special case among the regional centres. Its spatial influence is virtually a subset of a wider area of spatial influence of Bratislava. Bratislava is 6 times larger than Trnava and its economic potential is even more

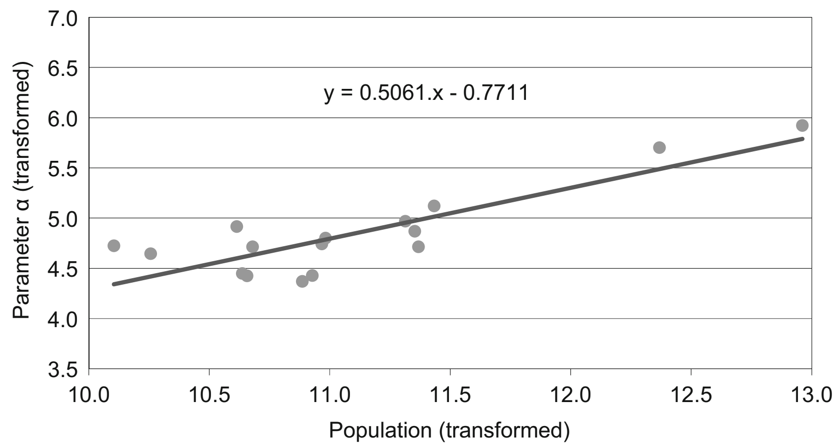

Fig. 10 Regression dependency model for parameter α_2 based on the population

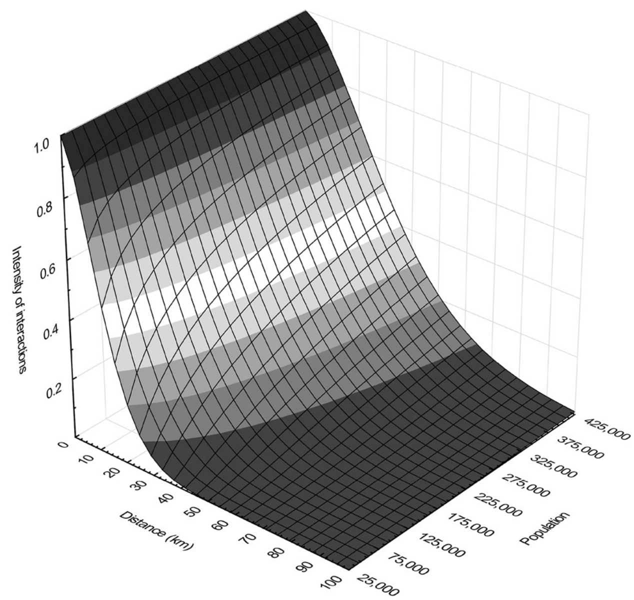

Fig. 11 Universal distance-decay function for daily travel-to-work flows to regional centres of Slovakia (dependent on the population)

significant in comparison to Trnava. Thus, Bratislava represents, in this case, a strong superior competing centre, which considerably decreases the spatial influence of Trnava. Therefore, the spatial influence of Trnava significantly lags behind its population size and this centre cannot be classified in any category in the resulting typology. Together with Komárno, Trnava has been left out of the regression analysis.

Regional centres of Slovakia can be classified into four groups according to the distance-decay functions, more precisely according to the relation between spatial influence and population size (Fig. 12):

(a) Macro regional centres Bratislava and Košice that have a specific position and spatial influence in the regional and settlement system of Slovakia. They are dominant centres for the whole territory of Slovakia (Bratislava) and eastern Slovakia (Košice), their spatial influence significantly exceeds all other centres in Slovakia.

(b) Regional centres whose spatial influence is adequate to their population size. This is a characteristic of almost all larger regional centres approximately at the level of regional capitals (Banská Bystrica, Martin, Prešov,Trenčín, Zvolen, Žilina). Nitra can also be included, even though its spatial influence is slightly lowered by the interaction effects of Bratislava and other western Slovakia "competing" centres. However, that is not as significant as the case of Trnava.

(c) Regional centres whose spatial influence is larger than their population size. This is particularly, a characteristic of regional centres localised in peripheral areas with a low population concentration and competition from other regional centres (Lučenec, Michalovce, Rimavská Sobota).

(d) Regional centres whose spatial influence is smaller than their population size. This is particularly a characteristic of regional centres localised in central areas with a high population concentration and competition from other regional centres (Nové Zámky, Poprad, Považská Bystrica, Prievidza).

With regard to the models introduced to geography for instance by Christaller (1933), the presented typology show a hierarchical organisation of the Slovak settlement system. Bratislava and Košice make up two individual hierarchical levels, the spatial influence of remaining centres basically depends on their population size, their hierarchical division is continuous rather than discontinuous. Physical geographical traits and other factors are responsible for the fact, that the spatial organisation of regional centres and their hinterlands is less regular in Slovakia in comparison with the territory analysed by Christaller, but also with the

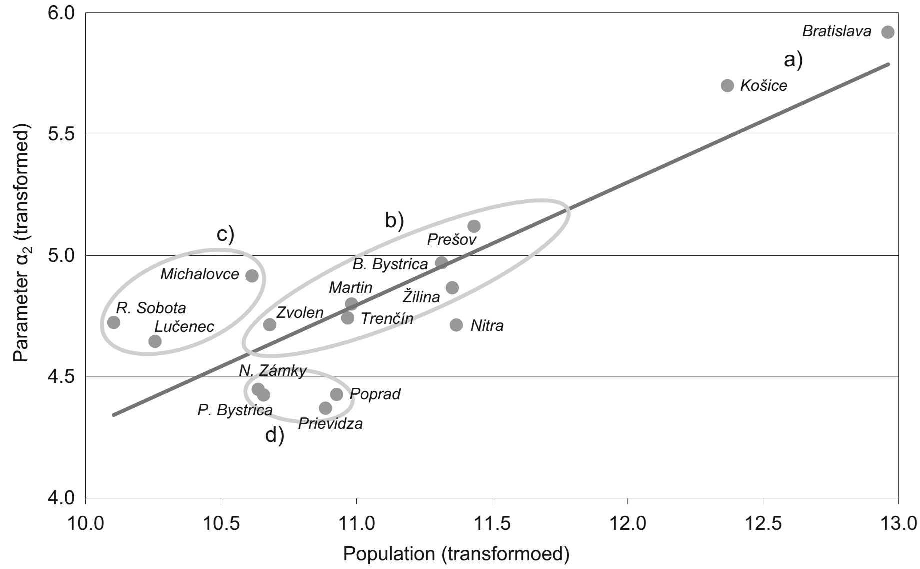

Fig. 12 Typology of regional centres of Slovakia according to their spatial influence

territory of neighbouring Czech Republic (Halás 2014; Klapka et al. 2014; Kraft et al. 2014). This shows in a way that original neoclassical models were too simplified and that new models of uneven spatial distribution of economic goods provide more realistic view of the spatial organisation, because they comprise a wider spectrum of input information including a market imperfection, role of monopoly, phenomenon of spatial agglomeration of economic activities etc. (Fujita et al. 2001). The issue of centrality, spatial organisation and hierarchy remains in the field of interest of regional science; for instance Taylor et al. (2010) explain the newer theory of central flows through the theory of central places.

4.4 Possibilities for application of distance-decay function in administrative arrangements

One of the possible applications of the distance-decay function is during changes and revisions to the administrative divisions of territories. If, at the lowest regional level of an administrative division (i.e. NUTS 4 level), the regions should have a nodal or functional nature, while at higher levels the rules for administrative divisions are not given unambiguously. The important factor for identification of administrative centres is the determination of their spatial influence, which is expressed, in the analyses discussed, either by the radius of influence S, or the α_2 parameter. Should potential regional capitals be identified (NUTS 3 regions), Trnava would have to be left out of the list of regional capitals according to the distance-decay function analysis. In contrast, some other regional centres, such as Michalovce, Martin, Zvolen, Rimavská Sobota, Lučenec or Poprad (according to the results of this paper and the data in Table 2; Figs. 9, 12), have the potential to be regional capitals. If pairs of regional centres located near each other (Banská Bystrica and Zvolen, Lučenec and Rimavská Sobota) are considered as options for the same territory, or as individual centres for different territories, the number of regional capitals and regional administrations is 11–13. However, the issue of regionalisation tasks and construction of administrative regions is much more complicated and the solutions would require much more detailed analyses carried out in several separate studies (e.g. according to the hierarchical level of target regions).

5 Conclusion

Even though regional and settlement systems are generally very complex and daily movements of population to regional centres are influenced by many factors, it is possible to quantify these movements and the changes in their intensities. The application in the territory of Slovakia has shown that this quantification has a relatively high information value. Distance-decay functions and analysis of their shapes have considerably helped the assessments. The bell-shaped function, i.e. a decreasing function with an inflexion point changing its curve from concave to convex, has appeared to be most suitable for the approximation of daily travel-to-work flows to regional centres. Except for minor exceptions (e.g. Poprad) the distance-decay functions for most regional centres in Slovakia have a very similar shape, they differ substantially only in the spatial influence of regional centres. This influence can be expressed by an area below the function curve, the so-called radius of influence S. The similarity of function shapes and dependency of the S parameter (radius of influence) on size characteristics of regional centres enables us to estimate the so-called universal distance-decay function according to a general indicator such as the population, for instance. This approach somewhat reduces the information value of the universal function, but according to statistical assessments it is still sufficiently relevant.

The analysis of the distance-decay functions has confirmed the unique dominance of Bratislava in the regional and settlement systems of Slovakia, and the dominance of Košice in the regional and settlement systems of eastern Slovakia. In all remaining regions, there are more level and regular relations between regional centres and their spatial influence depends on their size. This dependency is still higher in cases where the analyses have been carried out separately for core and peripheral areas. The only major exception is Trnava, whose position in close vicinity to dominant Bratislava significantly reduces its spatial influence. The spatial influence of Trnava is therefore, proportionally small with regard to its population size. Further operations with distance-decay functions for regional centres of Slovakia can be aimed at their practical spatial applications, particularly at the analysis possibilities for communication network planning and the identification of divides between spheres of influence of individual centres, and the ensuing regionalisation tasks or proposals and revisions to the administrative organisation of a territory.

On the basis of the preceding work (Halás et al. 2014), which analyses the distance-decay functions for regional centres of the Czech Republic, there are possibilities for comparisons and a generalisation of the results. The results have shown that the power exponential function is a suitable distance-decay function which approximates daily travel-to-work flows in both states, and that it has some significant advantages in comparison to the more complex Richards' function. The universal distance-decay function with a similar shape, given by the β_2 parameter, can be estimated on the basis of the power exponential function. For regional centres in the Czech Republic, the value of the β_2 parameter is 1.57, for regional centres in Slovakia it is almost the same; 1.59. Regarding the distinct social polarisation of the Slovak territory the universal function is based on population, while in the Czech Republic the number of job positions has proved to be more favourable. The already-mentioned polarisation means that higher reliability is reached in the case of Slovakia, where the dependency of the α_2 parameter on the population is expressed separately for core (the rich north-west) and peripheral areas (the poor south-east).

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