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Distance-decay functions for daily travel-to-work flows

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ABSTRACT

The non-homogeneity of geographic space brings about the processes that horizontally relate sections of geographic space, in transport geography referred to as spatial interactions. The distance separating different types of locations plays a crucial role in these interactions. Distance is the major factor that influences the values of interaction intensities. The question is how the intensities decrease with distance, since this decrease is usually not linear. This paper pursues the issues of the shape and parameters of the distance-decay functions based on daily travel-to-work transport movements, taking regional centres in the Czech Republic as the example. First the special distance-decay functions for individual regional centres are presented and discussed, followed by the expression of the universal distance-decay function approximating generally to the traits of the Czech settlement system and the nature of the interaction flows, i.e., travel-to-work. The expression of the universal function is based on the application of two easily accessible variables: population and number of jobs.

1. Introduction

Geographic space is not homogeneous and its constituent elements are distributed in an uneven manner. Their quantities and qualities vary across sections of geographic space, across locations and regions. This holds true both for the scalar, vertically organised characteristics and for the vector, horizontally organised characteristics of geographical space, the former being typical of homogeneous or formal regions, while the latter is connected particularly with heterogeneous or functional regions. This paper just pursues the latter case, since it is not primarily interested in the spatial distribution of geographical phenomena, but rather in the spatial interaction between sections of geographic space. Ullman (1980) refers to this interaction as a situation (or situational concept) wherein phenomena in one section have an influence on phenomena in another section.

In most cases involving the inherent heterogeneity of geographic space there is a natural tendency toward balance in the differences (or polarity) between locations and regions, by means of the aforementioned spatial interactions. This process has a physical basis within the scope of physical geography and can be illustratively witnessed in, for instance, the form of wind streams between areas with different air pressure values as they balance the original polarity in geographic space. Human geography sought inspiration from the physical sciences in this respect as early as the 1940s, resulting in the concepts of demographic gravitation (Stewart, 1948) and the principle of least effort (Zipf, 1947), both closely connected to the subject of spatial interactions to the present day. Spatial interactions in this case are expressed in the form of aggregated individual horizontal flows, mobilities, or contacts of persons, goods, finances, or information. Despite its inspiration in physical science, the issue presented in this paper is related to the study of human behaviour, particularly of transport movements, in space and time.

Human geography has been aware for a long time of the fact, sketched generally in the preceding lines, and reflected unintentionally in the behaviour both of particular individuals and of society (as an aggregation of these individuals), that there are different degrees of spatial or geographical attractiveness and utility (in their broadest senses within the range of human geography, including not only transport systems but also retail and service opportunities, economic activities, recreational opportunities, etc.) for different locations or sections of geographical space – regions of different hierarchical scales (see for instance the general introduction in Abler et al., 1972; Morrill, 1974 or Ullman, 1980). The spatial behaviour of individuals is influenced by their individual needs and by their efforts to optimise their spatial location and movements in the search for economic or social benefits, and it incorporates the possibility of choice and some psychological constraints, as in the perception of distances (Stouffer, 1940; Converse, 1949; Carrothers, 1956; Ullman, 1980; Taylor, 1983; Plane, 1984; Fotheringham, 1986; Stillwell, 1991; Johansson et al., 2002a;

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Timmermans et al., 2003). As such, spatial interactions considerably influence the spatial organisation of society and express the mutual relations between sections of geographic space, emphasizing the situational spatial context.

The paper has six sections. After this brief introduction the aims of the exploration are set and possible contributions to the state of the art suggested (Section 2). Section 3 then provides the concise theoretical background of the issue explored based on the literature overview. The method is described in detail in Section 4 and results are provided in Section 5. Section 6 includes some final remarks and generalisations.

2. Aims

The paper deals with a specific type of spatial interaction, daily travel-to-work flows, and attempts to analyse its relation to the distance and characteristics of geographical environment, particularly the settlement system. The intensity of spatial interaction depends primarily upon the distance separating two locations (regions), two other important factors influencing this intensity being intervening opportunities and the complementarity of locations. Taylor (1983) understands distance as a structural characteristic of a particular situation, which is easy to determine. The role of distance is expressed in simple form by Tobler's well known "first law of geography," which states that "everything is related to everything else but near things are more related than distant things" (Tobler, 1970, pp. 236).

With regard to transport geography the paper puts forward several contributions to the current state-of-the art of the discipline and human geography in general. It formulates the basis for the modelling of daily travel-to-work flows. This interaction represents diurnal transport movements of a particular regularity and intensity that are, unfortunately, statistically surveyed only once in ten years as part of a population census. However, the spatial pattern of daily travel-to-work flows changes more rapidly with respect to both economic conditions and development of transport infrastructure. If an up-to-date knowledge of potential transport flows and the provision of sufficient information for planning transport systems are to be maintained, the modelling of daily-travel-to-work flows appears urgent, because it is capable of simulating travel behaviour, in this case based on daily cycles.

The paper introduces the universal distance-decay function, based on both univariate and bivariate regression that is able to estimate daily travels in the hinterlands of regional centres with a sufficient goodness-of-fit and without the need for statistical data on horizontal flows, particularly on daily labour commuting. The data on population and/or number of jobs provide sufficient input variables. The function is complex rather than trivial but it still enables necessary generalisations. It is a compound power-exponential function with two parameters to be estimated and an inflexion point (see below Section 3). This form of the distance-decay function has been rarely found in the literature up to now. The paper introduces a new parameter, the so-called radius of influence (see below Section 4), which determines the spatial extent of hinterlands of regional centres; the parameter has a wider applicability apart from the transport geographical tasks. The paper suggests a universal value of the parameter β for the distance-decay function of the researched territory. Finally the paper provides calibrated individual distance-decay functions for all relevant regional centres of the researched territory, i.e. for the Czech Republic.

The main objective of the paper is therefore a construction of a distance-decay function for daily travel-to-work transport movements into the regional centres of the Czech Republic. Each city has its own specific conditions, and daily transport movements in its hinterland are determined by its location – both absolute and relative to the location of competing centres – and by its geographical context, particularly its historical one. Firstly we attempt to reach the best approximation of the distance-decay functions for each from the set of regional centres of the Czech Republic and then to provide a typology of the centres based on the shape of the curves of these functions. Secondly we attempt to express the universal distance-decay function for these regional centres generally (though of course less precisely) on the basis of simple and easily accessible variables, which in our case are the population and the number of jobs in the researched locations. Thirdly we would like to contribute to international comparison studies by presenting the state of a traditionally developed Central European country, though one affected by 40 years of Communist administration and central planning, and the transition to a market economy. In our opinion, the historical development of the Czech Republic presents the country as an interesting local example of variations in spatial interactions and spatial analyses (see Taylor, 1983; Fotheringham and Brunsdon, 1999).

The paper also has practical potential, as the correct estimation of distance-decay functions of daily travel-to-work flows will mitigate a dependency in general on population censuses (with their ten year interval) for providing the necessary data, and mitigate specifically the deteriorating quality of the travel-to-work data expected in the results of the 2011 census of the Czech Republic. We hope that the results of the paper could also contribute to the problem of spatial interactions modelling in relation to the employment of the values of the parameters controlling the measures of distance.

A concise insight into the character of the area under study should be provided to the reader as well. Central Europe has a long tradition in spatial analysis research (for instance Christaller, 1933 is the best example). The territory of the Czech Republic has a historically well-developed and stratified settlement system, similar for instance to that of Germany, and is quite homogeneous in terms of its physical geographical traits, with its borders along the chain of higher mountain ranges. This means that the space does not generally hinder horizontal flows, and that it is rather permeable for spatial interactions. Basic and generalised characteristics of the settlement system of the Czech Republic are sketched in Fig. 1, where the spheres of influence of the Czech micro-regional and higher centres have been ascertained by the application of the Reilly's model (Halás and Klapka, 2010). The western two-thirds of Czech territory, the historical region of Bohemia, are clearly dominated by the capital Prague as the natural centre of the whole republic. Other Bohemian centres appear hierarchically two levels lower than Prague. The eastern third, the historical regions of Moravia and Silesia, has a different, polycentric character with two prominent centres, Brno and Ostrava, each hierarchically one level lower than Prague.

3. Background

Besides distance, referred to above as the measure of spatial separation, the intensity of spatial interactions depends in general on the direction of the interaction, the qualities and quantities of locations serving as the origins and destinations of the interaction flows, and of course on the nature of the interaction being researched. As a general rule and through experience, it is claimed that interaction intensity decreases as distance increases. It should be noted here that interaction intensity usually increases with the "importance", "size", or "mass" (both quantitatively and qualitatively expressed) of the locations. The relation between interaction intensity and distance is a complex one. The main problem lies in quantifying the measure and shape of the decreasing function of the spatial interaction. A family of distance-decay functions (a term inspired by Wilson, 1971, 1974; Taylor, 1971b) is usually employed for these purposes; they are able to mediate this

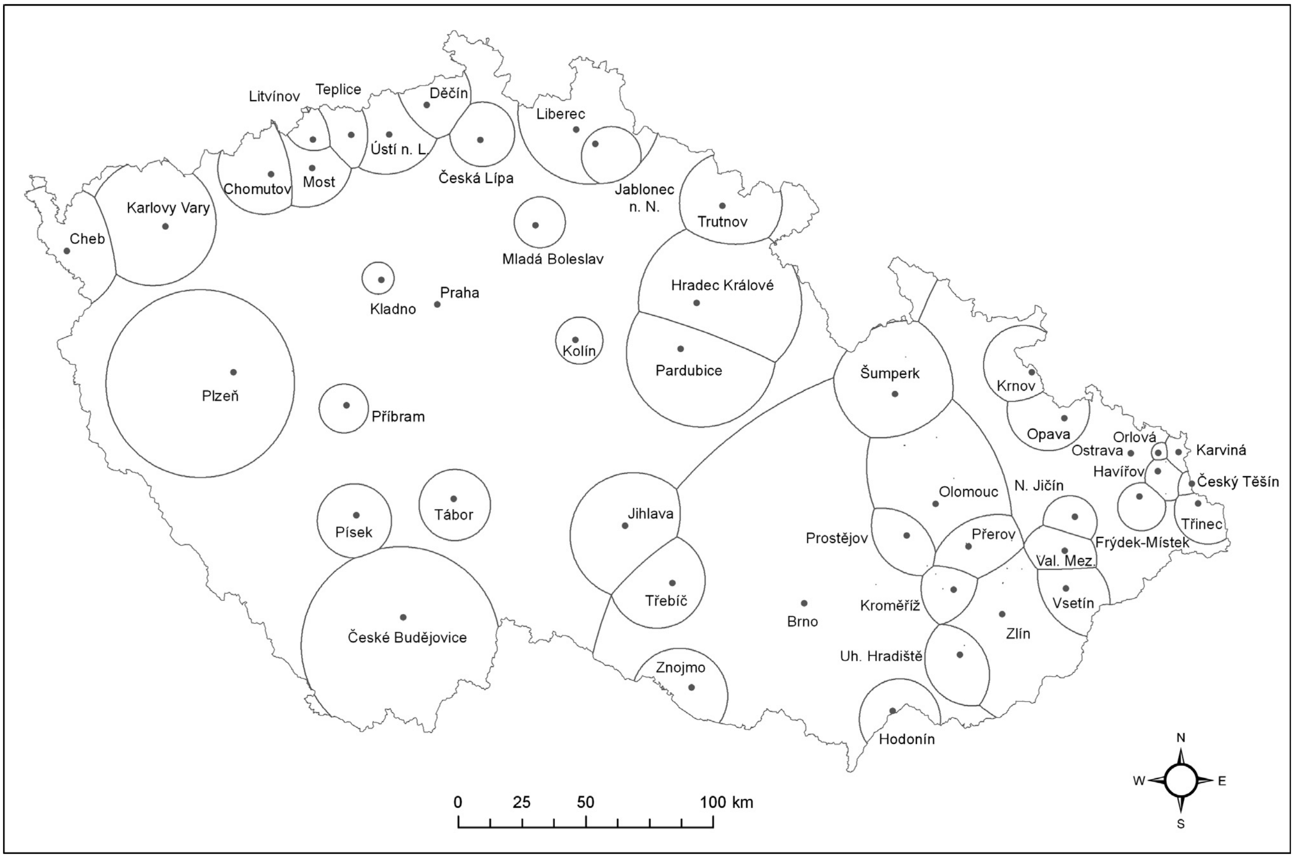

Fig. 1. Modelled spheres of influence in the Czech settlement system (towns with more than 25,000 inhabitants). *Source*: Halás and Klapka (2010).

decrease with a certain precision, empirically or graphically (Huff and Jenks, 1968).

It is generally agreed that the inverse distance-decay functions in most human geographical applications do not have a linear shape (Stewart, 1941; Goux, 1962; Taylor, 1971a, 1983; Johnston, 1973; Wilson, 1974; Alonso, 1978; Sheppard, 1978; Robinson, 1998; Haggett, 2001). Simple functions usually do not approximate well the observed and plotted data on a distance – interaction intensity graph, and so a family of more complex distance-decay functions should be employed instead, according to the nature of the data being researched (Taylor, 1983; Robinson, 1998). Three methods can be applied (Taylor, 1983): the use of several linear functions to approximate the plotted data, the use of a polynomial curve, or the transformation of the original interaction data. The last method has been used most frequently and it provides the following functions: square root exponential, exponential, normal, Pareto and log-normal (see more in Goux, 1962).

The Pareto function has played a major role in the application of distance-decay in human geographic research as it is a direct analogy to Newtońs law of gravitation. As such it has been employed in illustrative cases of spatial interaction modelling (Isard, 1960 (and later Isard et al., 1998), Haggett, 1965, 2001; Abler et al., 1972 etc.), though its suitability for more specific tasks has been questioned, for instance by Hägerstrand (1962), Taylor (1971a), Cliff et al. (1974), Wilson (1974), Fotheringham (1981), De Vries et al. (2009). Most of these critical works have suggested searching instead for a distance-decay function dependent on the input data, on the research questions, or on the territory under study. We fully comply with their findings in this paper. Nevertheless, some relatively recent studies have still empirically confirmed the applicability of the Pareto function and its modifications, for instance Aubigny et al. (2000), Grasland and Potrykowska (2002), an officially unpublished working paper on the geographic potential model (Grasland, 1996), our own recent study on the regionalisation of the Czech Republic using a variant of Reilly's model (Halás and Klapka, 2010), the study applying the gravity model to the Vysočina region in the Czech Republic (Kraft and Blažek, 2012), and the paper on the modelling of the intra-urban shopping trips in the city of Olomouc, Czech Republic (Klapka et al., 2013).

However, in all three aforementioned instances of dealing with the non-linear character of the original plotted interaction data (i.e., the use of several linear functions, polynomial functions, and original data transformation), the data are subject to procedures which unfortunately bring about a certain degree of information loss, and the functions do not fit the data distribution well in certain research tasks. Therefore, in particular cases such as the one presented in this paper, it is necessary to define and construct even better fitting functions, based on original interaction data, since the aforementioned ones still possess a rather general character. Our argument as to how to reach a higher level of goodness-of-fit is provided in Section 4.

Apart from traditional forms of distance-decay functions as discussed above, a family of more complex functions can be found in the relevant literature. These distance-decay functions have a bell shape and an inflexion point, and they are controlled by two parameters to be estimated in most cases, such as Tannerś function, March́s function, Weibull function, squared Cauchy function or Box–Cox function (see for instance Taylor, 1971a; Openshaw and Connolly, 1977; Tanner, 1978; Openhshaw, 1998; Nakaya, 2001; Celik, 2004; Celik and Guldmann, 2007; Langford et al., 2012 or Martínez and Viegas, 2013). Richardś function (touched upon for instance by Martínez and Viegas, 2013) applies up to four parameters. It better approximates individual instances but its use in the formulation of a generalised or universal distance-decay function is rather limited. A function based on two variables, which is a similar concept to one of the functions proposed in this paper, has been formulated by Mozolin et al. (2000), who apart from distance, applied the number of employed in the centre as well, and their function has been presented in the form of a 3D diagram.

The form of the distance-decay function can be influenced by a travel mode or a travel purpose. Firstly some notes regarding the travel mode and its relation to the distance-decay function should be made. As the use of a particular travel mode has its specifics (purpose of travel, duration, cost etc.) these are certainly reflected in the shape of a particular distance-decay function. Zhao et al. (2003) discuss a walking distance effect on public transport accessibility, favouring the use of the exponential function in this specific case. Gutiérrez et al. (2011) analyse the more complex issue of ridership that comprises more travel modes within the intra-urban environment and they apply a weighted distance decay concept. A similar problem is tackled by Mamuna et al. (2013) using a time distance.

As this paper pursues a specific, though very frequent, instance of a spatial flow (i.e., travel-to-work movements), we will mention particularly some relatively recent works pertaining to the issue of distance-decay functions in this field of study (i.e. residence – workplace relations). Johansson et al. (2002b) used time distance for labour commuting in their study and confirmed the non-linear character of the decrease in interaction intensity with increasing distance. Ubøe (2004), although using the term ''deterrence function'' instead, arrived at the same conclusion by applying aggregation gravity models to the travel-to-work flows. O'Kelly and Niedzielski (2009), discussing among other things the values of the distance-decay parameter in functions relating to the size of a commuting centre, proved the dependency of the modelled shape of the distance-decay function (and thus of the extent of the commuting hinterland) on the quantitative characteristics of the centres of the settlement system.

Heldt Cassel et al. (2013) discuss the role of time distance and the socio-demographic characteristics of the job seekers on their willingness to travel to work. Although they do not transpose their findings into the formulation of a distance-decay function, their work draws attention to the interesting possibility of the extension of spatial interaction models (see for instance Martínez and Viegas, 2013). Cheng and Bertolini (2013) address the job accessibility in urban centres using more detailed data on individuals and a simple distance-decay function, which however takes into account the type of worker and the travel model. A residential location choice model with respect to the distance to workplace is discussed by Ibeas et al. (2013).

4. Method

Data on migration, particularly on labour and school commuting, provide crucial information on the spatial mobility of the population and on spatial interactions. Labour commuting (i.e., travel-to-work), as a basic foundation for regionalisation tasks, was first surveyed in Czechoslovakia in the 1961 census. Since that time, we have had data on the main directions of commuting flows but only at ten-year intervals. Other data on spatial interactions (e.g., the number of transported persons, shopping centre visitation, etc.) are very difficult to acquire and in numerous cases are secured as trade secrets. This paper relies upon the 2001 census' daily labour commuting data. If the practical distance of daily movement is limited, the required distance-decay functions must either reach the zero value at a certain distance from a centre or else approach zero (i.e., the x axis) asymptotically.

The distance-decay function for a particular centre has been constructed in the following way: the x axis shows the distances (in km) from various municipalities to the given centre, while the y axis gives the portion of daily commuters from these municipalities into the given centre out of the total number of local daily out-commuters. These values occur in the interval between 0 and 1 (or 0–100%) and are referred to as the interaction intensity. The interaction intensity in the centres is not measurable of course; we posit, then, that the value of the interaction intensity in the centre, i.e. at the zero distance, is 1 or 100%. Each municipality is represented by a single point on a graph. In relation to the location of these points (municipalities) in the graph coordinates, we attempt to express the optimal distance-decay functions for the particular centre. The graph values are not transformed, since in this form the information capability of the distance-decay function is the greatest (e.g., for further three dimensional expressions, or for other applications).

The construction of the distance-decay function requires the distances from the municipalities to the centres. In the model, we have used as road distances the fastest variants computed by the route planner of the Škoda Auto Company (see www.skoda-auto.com/cz), given to the nearest 0.1 km. Although the data have been processed for the 144 regional centres defined in the human geographical regionalisation of the Czech Republic (Hampl, 2005), that have been also used for the identification of the transport hierarchy of the regional centres in the Czech transport geographical research (most recently in Kraft, 2012), detailed analysis has shown that smaller, micro-regional centres did not produce a sufficient number of interactions for an adequately reliable distance-decay function. Therefore, we have restricted our study to centres with more than 25,000 inhabitants. This level has been chosen since it includes (with a sufficient reserve) all centres with at least a theoretical potential to become centres of mezzo-regions, i.e. of the regions at approximately NUTS 3 level (Halás and Klapka, 2010).

A methodological note on the Euclidian expression of a distance should be made here. Time or cost units represent another approach to the distance problem (see for instance Zhao et al., 2003; Heldt Cassel et al., 2013; Mamuna et al., 2013; Martínez and Viegas, 2013). However, these expressions were not used in the paper because the need to differentiate between transport modes would arise and we would face the problem of a shortage of sufficient data. Furthermore, several transport modes can be used by individuals on their travels, thus we consider the kilometric distance as the least problematic with respect to potential errors in our estimates, particularly when forming the hinterlands of the regional centres and estimating the radius of influence S, which would be unsuitably affected by differences in time distance for various parts of the researched territory and for various travel modes.

The first tests were applications of standard functions with no inflection point, such as power, exponential, log-normal, and Pareto functions, these being the most frequently used for the interaction approximation (see for instance Goux, 1962; Taylor, 1983; Robinson, 1998 etc.). However, in no case do these functions accurately reflect the daily labour commuting movements. As we see in the instance of Prague, the function either does not fulfil the theoretical postulate of reaching the point [0; 1], or else it runs totally outside the main point cluster (Figs. 2 and 3). The application of these functions to the other centres provided the same results.

According to the distribution of points on the graph, it is quite clear that an application of the bell-shaped function – i.e., a decreasing function with an inflection point, changing its curve from concave to convex and beginning at the point [0; 1] – will be more favourable. In order to express such a function, two factors are crucial: the extent of the regional centre's influence and the manner in which this influence decreases. Therefore at least two variable parameters have to be employed in order to express the optimal distance-decay function for the regional centre. The simplest function obeying these conditions is a compound power-exponential function in the form:

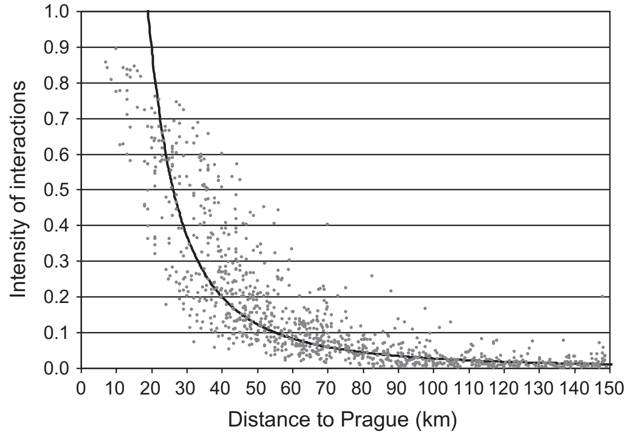

Fig. 2. Distance-decay function (power) for daily travel-to-work to Prague.

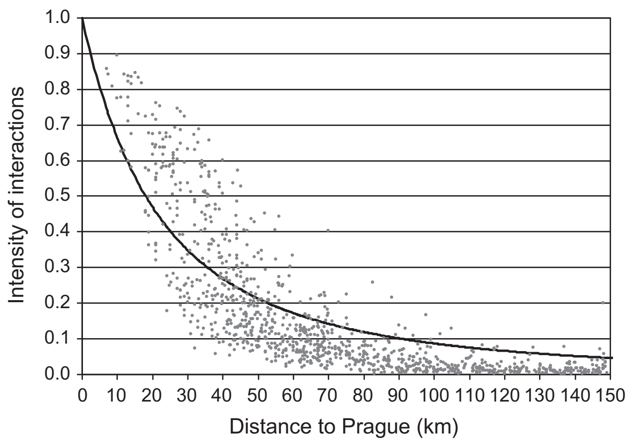

Fig. 3. Distance-decay function (Pareto) for daily travel-to-work to Prague.

$$f(d) = exp(-\alpha \cdot d^{\beta}),$$

where d is the distance from the centre; α, β are parameters, $\alpha > 0$, $\beta > 0$. A similarly shaped distance-decay function was presented by Martínez and Viegas (2013), while in its most complicated form this function had four parameters. The advantage of the function proposed in this paper is that it sufficiently approximates the intensity decrease of the daily travel-to-work flows using a simpler form and only two parameters. Unlike the aforementioned work (Martínez and Viegas, 2013), the x axis represents the Euclidean distance, not the time distance, since the daily travel-to-work flows are produced by different transport types and the conversion of the Euclidean distance to the time distance would require a weighting of the transport types, and thus too many subjective inputs would enter the function.

The important indicator expressing the extent of the influence of the regional centres is the area S below the curve. In a standard case it could be expressed by a certain integral:

$$S = \int_0^{\infty} f(d)dd,$$

But if the selected function is transcendental, there is an antiderivative function counterpart, which cannot, however, be expressed by an elementary function of a final shape. Therefore we have applied the approximative rectangular method with a calculation step of 10^{-5} km (i.e., 1 cm). S expresses the theoretical distance of the 100% extent of a regional centre (Fig. 4), hereafter called the "radius of influence" (because we employ road distances, the potential shape of the area of influence of the regional centre will not necessarily be an exact circle). It is worth mentioning here

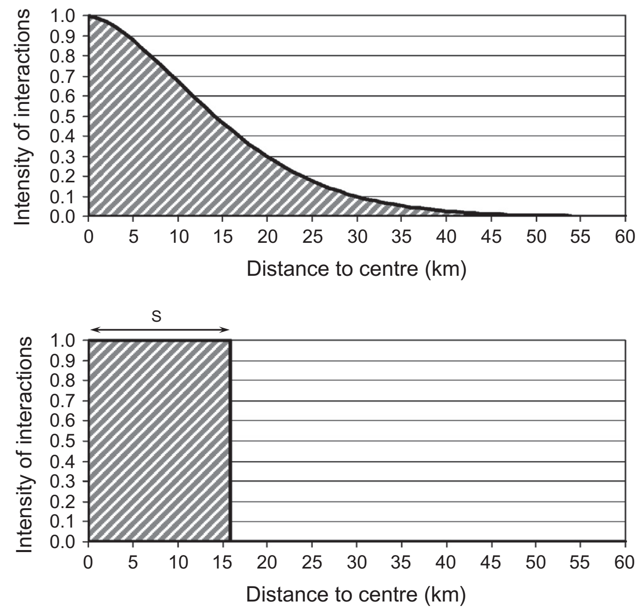

Fig. 4. Graphical explanation of radius of influence S for regional centres.

that the S value sets the radius of influence and the area of influence of the centre but it does not express the population size of the area.

In order to carry out the typology of distance-decay functions, it is necessary to understand the behaviour of the compound power-exponential function when adjusting its parameters α and β. When increasing the α parameter the extent of the centre's influence is considerably reduced without a significant change in the shape of the curve. The same holds true for the β parameter – increasing its value causes a considerable reduction in the extent of the centre's influence without significantly changing the shape of the curve; only the position of the inflection point is slightly lowered in the graph (Fig. 5). The α and β parameters are mutually dependent, while individually they are minimally dependent on the size of the centre, with more significant dependency on the size of the centre identifiable only when they are combined. We can claim that the α and β parameters control the slope of the function, which is best witnessed when preserving the constant radius of influence S. If the value of the β parameter is higher, for the constant radius of influence S, the interaction intensities are relatively high in the closest hinterland of the centre, but at a certain distance from the centre they begin a sharp decrease; if the value of the β parameter is lower the interaction intensities begin their decrease from the centre sooner, but their later decrease is gentler (Fig. 6).

A further methodological procedure for the typology and construction of a universal distance-decay function is directly connected with the particular results that show distance-decay function shapes for particular centres. Therefore the method will

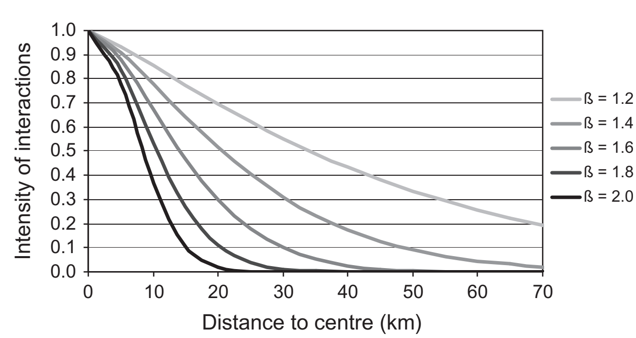

Fig. 5. Distance-decay functions for a constant parameter α.

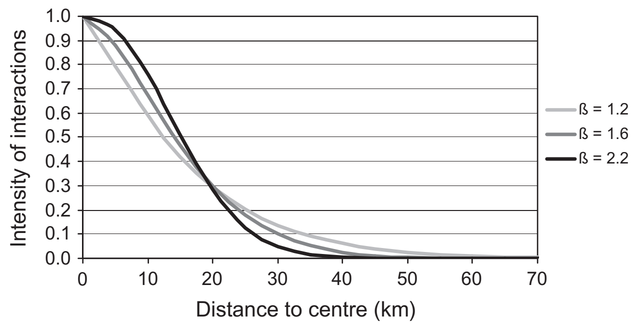

Fig. 6. Distance-decay functions for a constant radius of influence *S*.

be described, together with a presentation of the results in the following paragraphs.

5. Results

5.1. Basic distance-decay function

Travel-to-work flows from all relevant municipalities to a regional centre have been used to calibrate the basic (i.e. individual) distance-decay functions. Their resulting shapes and parameters are presented in Table 1(the regional centres are sorted by the newly introduced parameter, the radius of influence S); functions for the most important centres are graphically presented in Fig. 7. All parameters (Table 1) reflect with relative precision the settlement structure of the Czech Republic and the general character of its geographical environment (for more discussion see Section 6).

Out of two parameters controlling the shape of the basic distance-decay functions, the β parameter proves more stable (minimum 0.54, maximum 2.66). The α parameter shows a minimum of 0.0007, and a maximum of 0.7711. It provides another reason for the choice of the β parameter as the basis for the formulation of the universal function (see below). The relative stability of the β parameter rests in the character of the interactions, which are daily travel-to-work flows, and that tends to produce similarly shaped functions. However, both parameters are mutually dependent.

5.2. Typology of distance-decay functions

Considering the mutual dependency of the α and β parameters, just one of them must be employed in the typology of the distance-decay function for the centres. We have selected the β parameter,

Table 1
Parameters of distance-decay functions for daily travel-to-work to Czech regional centres.

Centre	Population (2010)	Parameter α	Parameter β	Coefficient of determination	Mean distance	Radius of influence *S*
Prague	1,249,026	0.0110	1.2855	77.2	48.3	30.9
Brno	371,399	0.0036	1.6596	84.9	27.8	26.5
České Budějovice	94,865	0.0039	1.7355	88.2	17.8	21.8
Plzeň	169,935	0.0152	1.3428	78.7	19.3	20.7
Znojmo	34,725	0.0630	0.9236	59.2	14.9	20.7
Ostrava	306,006	0.0080	1.5506	65.4	20.7	20.2
Mladá Boleslav	44,750	0.0075	1.6058	79.9	18.3	18.9
Hradec Králové	94,493	0.0059	1.7034	75.2	20.6	18.2
Jihlava	51,222	0.0037	1.8558	80.2	17.3	18.2
Prostějov	45,324	0.0267	1.2253	71.4	11.8	18.0
Třinec	37,405	0.0280	1.2137	44.1	12.9	17.9
Liberec	101,625	0.0215	1.3226	68.7	24.4	16.8
Karlovy Vary	51,320	0.0703	0.9558	53.4	15.8	16.4
Písek	29,949	0.0087	1.6466	74.6	16.4	16.0
Kroměříž	29,027	0.0655	0.9884	46.1	12.2	15.8
Olomouc	100,362	0.0065	1.7519	84.1	18.0	15.8
Tábor	35,484	0.0577	1.0347	61.9	14.6	15.5
Krnov	25,059	0.0376	1.1729	62.2	14.9	15.5
Zlín	75,714	0.0101	1.6236	79.2	17.2	15.2
Vsetín	27,558	0.0543	1.0670	62.3	14.3	15.0
Opava	58,440	0.0084	1.6972	79.5	13.9	14.9
Kolín	30,935	0.0375	1.1910	62.5	10.8	14.8
Pardubice	90,077	0.0347	1.2189	76.0	18.3	14.8
Příbram	34,217	0.0716	0.9854	72.3	12.6	14.6
Cheb	34,626	0.0136	1.5432	74.2	19.6	14.6
Chomutov	49,795	0.0401	1.1807	81.1	11.2	14.4
Třebíč	38,156	0.0480	1.1276	71.0	13.4	14.1
Přerov	46,254	0.0189	1.4516	77.3	11.5	14.0
Ústí nad Labem	95,477	0.0007	2.6615	83.2	19.2	13.6
Uherské Hradiště	25,551	0.2818	0.5858	63.1	10.2	13.4
Trutnov	31,005	0.0224	1.4431	59.5	15.0	12.6
Česká Lípa	38,104	0.0532	1.1374	69.2	16.5	12.6
Děčín	52,260	0.0058	1.9559	87.1	18.3	12.3
Most	67,518	0.0123	1.6864	81.7	16.9	12.1
Hodonín	25,526	0.0119	1.7090	78.7	12.9	11.9
Šumperk	27,492	0.0509	1.2049	72.2	11.3	11.1
Jablonec nad Nisou	45,328	0.0245	1.4866	84.9	9.9	11.0
Valašské Meziříčí	27,176	0.0157	1.6610	62.3	13.8	10.9
Kladno	69,938	0.1455	0.8657	55.8	11.0	10.0
Nový Jičín	25,862	0.0499	1.2906	83.0	13.4	9.4
Frýdek-Místek	58,582	0.2205	0.7511	64.7	13.3	8.9
Teplice	51,208	0.1028	1.0346	79.6	9.7	8.9
Litvínov	27,533	0.1419	0.9443	71.2	10.9	8.1
Karviná	61,948	0.0534	1.3540	84.1	13.9	8.0
Český Těšín	25,499	0.2465	0.8654	74.7	15.2	5.4
Havířov	82,896	0.3557	0.8263	76.7	10.8	3.9
Orlová	32,430	0.7711	0.5432	74.2	9.4	2.8

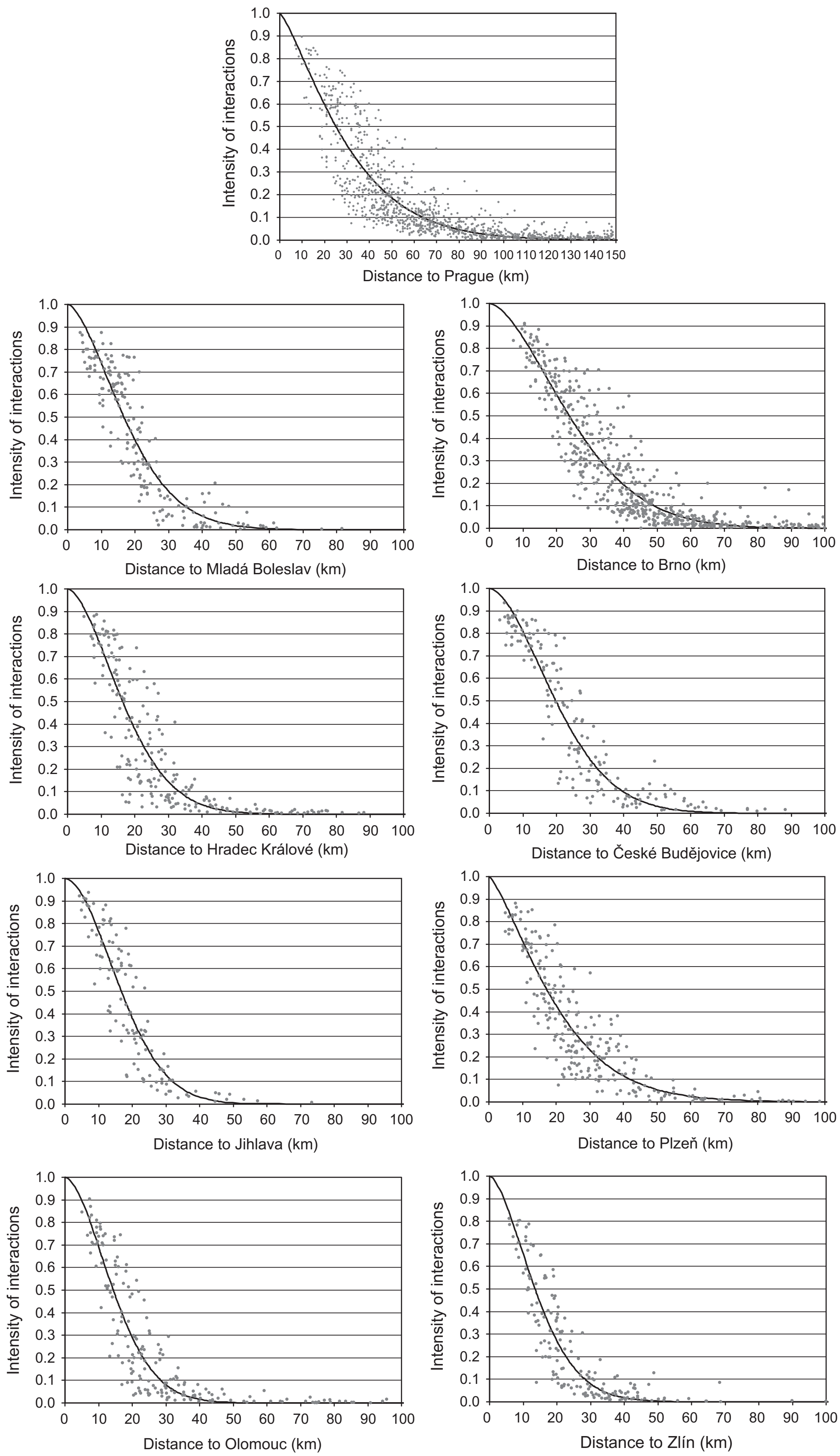

Fig. 7. Optimal distance-decay function for daily travel-to-work to Czech regional centres.

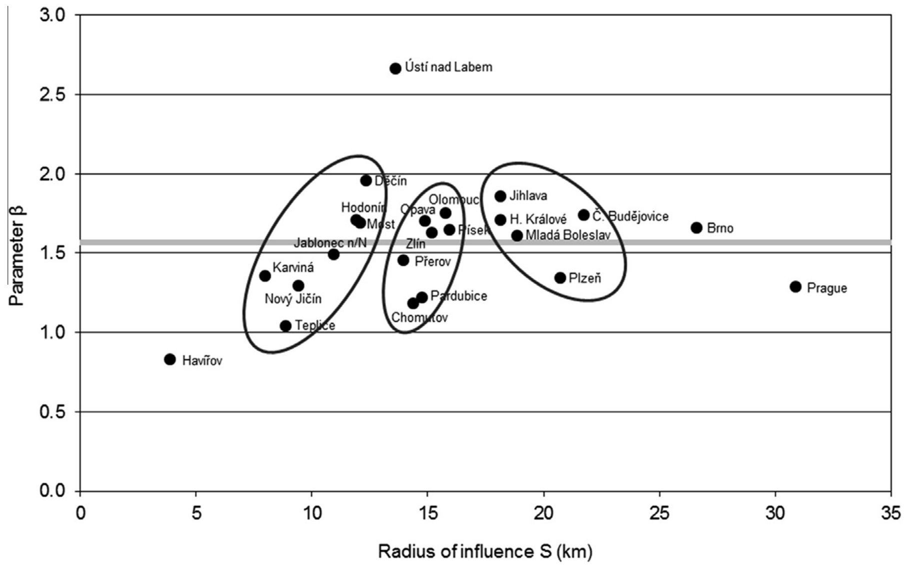

Fig. 8. Typology of Czech regional centres according to parameters of distance-decay functions for daily travel-to-work.

since it possesses almost zero dependency on the size of the centre and better controls the shape of the curve, including the position of the inflection point. Thus, in the resulting typology the change in the shape of the curve is determined by the β parameter. The extent of regional influence of the centres is assessed by the most representative parameter, which for this purpose is the radius of influence S. The resulting typology relies upon the relevant parameters β and S (Fig. 8). In order to ensure high information value, we have, in this phase, employed only centres for which distance-decay functions had a higher value of the coefficient of determination (greater than 0.75).

The β parameter value oscillates between 1 and 2 in most cases. The extreme case is Ústí nad Labem, with the parameter exceeding the value of 2.6. It means that Ústí nad Labem has, in its closest hinterland, quite strong interactions (i.e., most out-commuters from the immediate hinterland work in Ústí nad Labem), but that the interaction intensities sharply decrease with distance. This is documented as a 3D model in Fig. 9; when shown in comparison to Pardubice (a city with a similar size and value of radius of influence S), these cities differ considerably in the value of the β parameter. In the case of Pardubice the sharp decrease in the hinterland extent occurs immediately but these lower interaction intensities are retained over a longer distance.

Though these two cities are similar regarding their population, they differ in two factors that, we assume, generally influence the shape of distance-decay functions, i.e. a relative position in the settlement system, and particularly the transport type which serves as a medium for daily travel-to-work flows. In this respect quality railway transport favours the receivable interaction intensities for

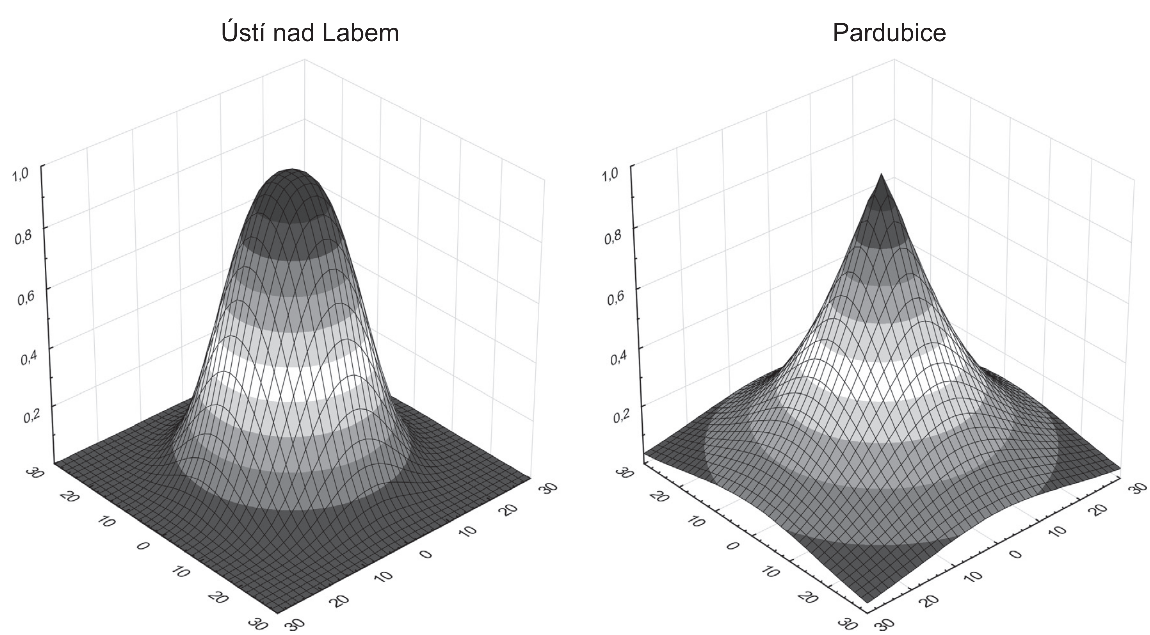

Fig. 9. 3D model of distance-decay function for daily travel-to-work to selected regional centres.

a longer distance from a centre in comparison with other transport types (see for instance Pardubice in Fig. 3 with an outstanding railway connection to its hinterland) and thus contributes to the formation of more extensive hinterlands.

As macro-regional centres, Prague and Brno occupy a special position and cannot be classified into any type with regard to their hinterlands. While the variance of the β parameter for other centres is not significant, these centres can instead be classified according to the extent of their hinterlands. Not only the population size of a centre, but also the other criteria described above (i.e., economic hinterland of a centre, competition and position of neighbouring centres, etc.) are decisive. A special example which differs from other centres is Havířov, which, thanks to the dominance of its housing function and the absence of job positions, has only a minimal hinterland.

Prague and Brno excluded, the other regional centres can be divided into three groups according to the radius of influence S of daily interactions:

(a) centres with a larger regional influence according to daily commuting flows, their radius of influence reaching 18–22 km (České Budějovice, Plzeň, Mladá Boleslav, Hradec Králové, Jihlava),
(b) centres with a medium regional influence according to daily commuting flows, their radius of influence reaching 14–18 km (Písek, Olomouc, Zlín, Opava, Pardubice, Chomutov, Přerov),
(c) centres with a smaller regional influence according to daily commuting flows, their radius of influence reaching 8–14 km (Děčín, Most, Hodonín, Jablonec nad Nisou, Nový Jičín, Teplice, Karviná).

If we only take into account the frequencies in the groups, the number of centres with a larger regional influence (a), plus Prague and Brno, will give us approximately the adequate number of NUTS-2 regions, according to the EU population classification. In the category of centres with a medium regional influence (b) we have not only potential centres of NUTS-3 regions, but also centres that have no ambition to become NUTS-3 centres (e.g., Písek, Přerov, etc.). This disparity is caused by the fact that the designation of NUTS-2 and NUTS-3 regions (it is the level of regional self-government in the Czech Republic) and the selection of their centres should not be based primarily on daily commuting flows.

5.3. Universal distance-decay function

The aforementioned results have only determined specific distance-decay functions for each centre individually, or been used for constructing their typology. Given that the variance of the β parameter is low between the optimal functions, we will attempt to construct a universal function by setting a constant value for this parameter. This constant has been reached by calculating a weighted average of the β parameter for all centres examined; weighting has been determined by the values of the coefficient of determination. Based on this calculation, the universal β parameter, henceforth referred to as β_2, has a value 1.57 (this value is presented in Fig. 8). Optimal values of the α_2 parameter for $\beta_2 = 1.57$ are presented in Table 2. The values of the coefficient of determination for individual centres testify that these functions do not significantly differ in their information value from the optimal functions presented in Table 1, and we claim that this construction of the universal distance-decay function has justification.

Table 2
Parameters of universal distance-decay function for daily travel-to-work to Czech regional centres.

Centre	Parameter		Coefficient of determination
	α_2	β_2	
Prague	0.0039	1.57	76.0
Brno	0.0050	1.57	84.8
České Budějovice	0.0066	1.57	88.0
Plzeň	0.0075	1.57	77.9
Znojmo	0.0098	1.57	47.7
Ostrava	0.0075	1.57	65.4
Mladá Boleslav	0.0083	1.57	79.9
Hradec Králové	0.0088	1.57	75.0
Jihlava	0.0086	1.57	79.4
Prostějov	0.0101	1.57	68.0
Třinec	0.0103	1.57	41.8
Liberec	0.0102	1.57	67.8
Karlovy Vary	0.0123	1.57	45.6
Písek	0.0108	1.57	75.5
Kroměříž	0.0139	1.57	37.2
Olomouc	0.0110	1.57	83.8
Tábor	0.0126	1.57	54.1
Krnov	0.0128	1.57	59.4
Zlín	0.0118	1.57	79.2
Vsetín	0.0137	1.57	55.4
Opava	0.0120	1.57	79.3
Kolín	0.0140	1.57	59.7
Pardubice	0.0130	1.57	73.8
Příbram	0.0151	1.57	60.6
Cheb	0.0126	1.57	74.2
Chomutov	0.0142	1.57	78.2
Třebíč	0.0150	1.57	65.8
Přerov	0.0139	1.57	77.0
Ústí nad Labem	0.0134	1.57	79.4
Uherské Hradiště	0.0266	1.57	–
Trutnov	0.0159	1.57	59.3
Česká Lípa	0.0171	1.57	65.4
Děčín	0.0156	1.57	86.3
Most	0.0167	1.57	81.6
Hodonín	0.0174	1.57	78.4
Šumperk	0.0205	1.57	70.0
Jablonec nad Nisou	0.0199	1.57	84.7
Valašské Meziříčí	0.0198	1.57	62.2
Kladno	0.0282	1.57	39.2
Nový Jičín	0.0253	1.57	81.8
Frýdek-Místek	0.0308	1.57	35.7
Teplice	0.0312	1.57	73.7
Litvínov	0.0335	1.57	61.0
Karviná	0.0325	1.57	83.4
Český Těšín	0.0508	1.57	49.1
Havířov	0.0911	1.57	55.9
Orlová	0.1643	1.57	–

The fact that the β_2 parameter is a constant and the α_2 parameter is a variable means that we acquire values of the α_2 parameter significantly dependent on the size characteristics of the regional centres. This dependency can be determined by a regression model. By a correct transformation (logarithmic) to the linear regression model of the α_2 parameter (or by a transformation of the population and jobs data), we acquire a relationship expressed by a linear function (Figs. 10 and 11). When expressing the linear dependency of the transformed α_2 parameter and the transformed population numbers, we generate a correlation coefficient of 0.81; for the linear dependency of the transformed α_2 parameter and the number of jobs, we acquire a correlation coefficient of 0.84. (Note: the model has not taken into account regional centres with significantly deformed hinterlands, i.e. centres with another centre at least four times larger in terms of population within 30 km.)

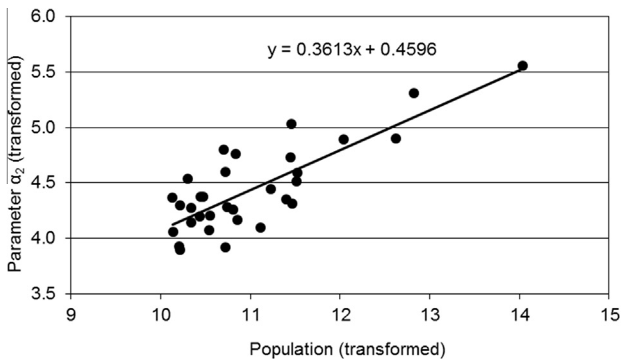

Fig. 10. Regression model of the dependence of the α_2 parameter on population.

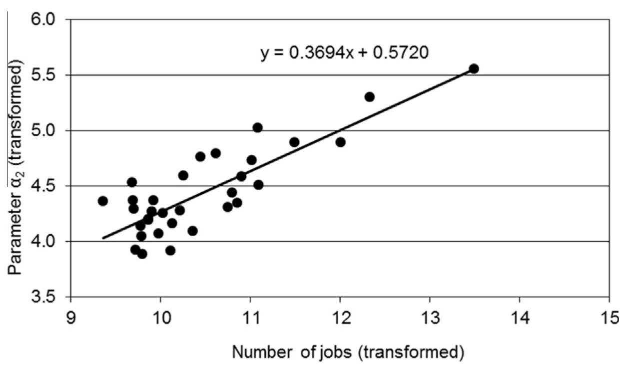

Fig. 11. Regression model of the dependence of the α_2 parameter on number of jobs.

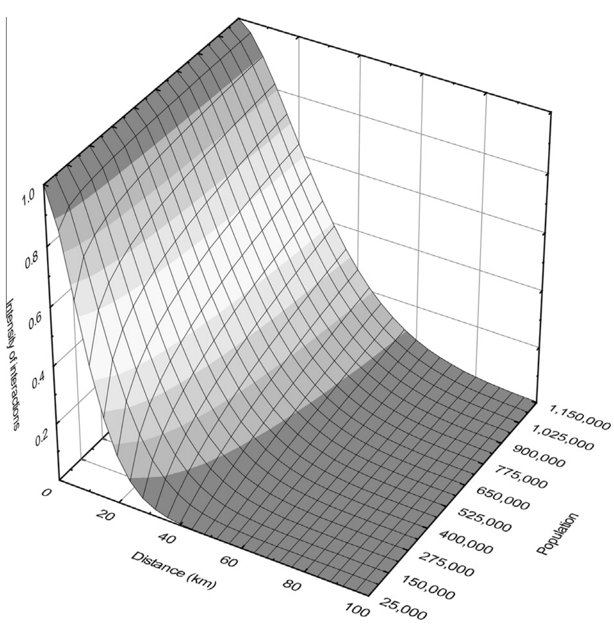

Fig. 12. Universal distance-decay function for daily travel-to-work to Czech regional centres (dependent on population).

Based on these facts, we can express the universal distance-decay function for regional centres by simple data that are easily acquired: the population of the regional centre or the number of jobs in the regional centre, whichever has a greater information value. The optimal function expressing the changes in the interaction intensities dependant on the distance from a regional centre and the population of the centre is[1]:

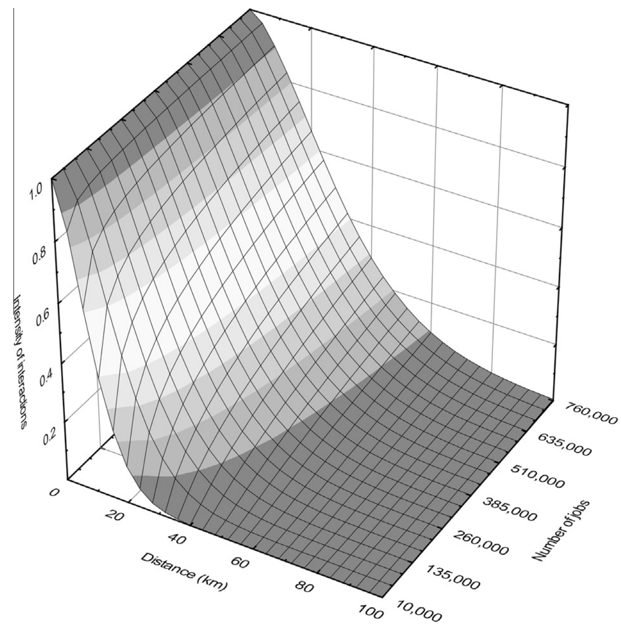

Fig. 13. Universal distance-decay function for daily travel-to-work to Czech regional centres (dependent on number of jobs).

$$f(d,i) = exp(-0.6316 \cdot i^{-0.3613} \cdot d^{1.57}),$$

when i is the population of a regional centre. The optimal function expressing the changes in the interaction intensities dependant on the distance from a regional centre and the number of jobs in a regional centre is[2]:

$$f(d,j) = exp(-0.5644 \cdot j^{-0.3694} \cdot d^{1.57}),$$

when j is the number of jobs in a regional centre. In both cases we have a function in two variables, and graphs of these functions are presented in Figs. 12 and 13.

The distance-decay function can be expressed analogically by the combination of the population and number of jobs. This dependency can be expressed by a bivariate regression model. Similar to the preceding case a model of bivariate linear regression dependent on the α_2 parameter transformations (or the transformations of both population and number of jobs) can be formulated as:

$$z = -0.3132x + 0.6738y + 0.8361.$$

The value of the multiple correlation coefficient r_{zxy} is 0.85 in this case. It means that this model estimates the dependency better than when the dependency is expressed only by the number of jobs. However, the difference is not significant. In a graphical expression we present both dependency plotted by transformed parameters (linear) and dependency plotted by untransformed parameters (power model – Fig. 14). Only regional centres larger than 120 thousand inhabitants are plotted on the graph of untransformed

[1] See: Fig. 10

$$y = 0.3613x + 0.4596;\ y = -\ln(\alpha_2);\ x = \ln(i)$$
$$-\ln(\alpha_2) = 0.3613 \cdot \ln(i) + 0.4596 \Rightarrow \alpha_2 = \exp(-0.3613 \cdot \ln(i) - 0.4596)$$
$$\Rightarrow \alpha_2 = 0.6316 \cdot i^{-0.3613}$$
$$f(d) = \exp(-\alpha_2 \cdot d^{\beta_2}) \Rightarrow f(d,i) = \exp(-0.6316 \cdot i^{-}0.3613 \cdot d^{1.57})$$

[2] Procedure is analogical to footnote 1.

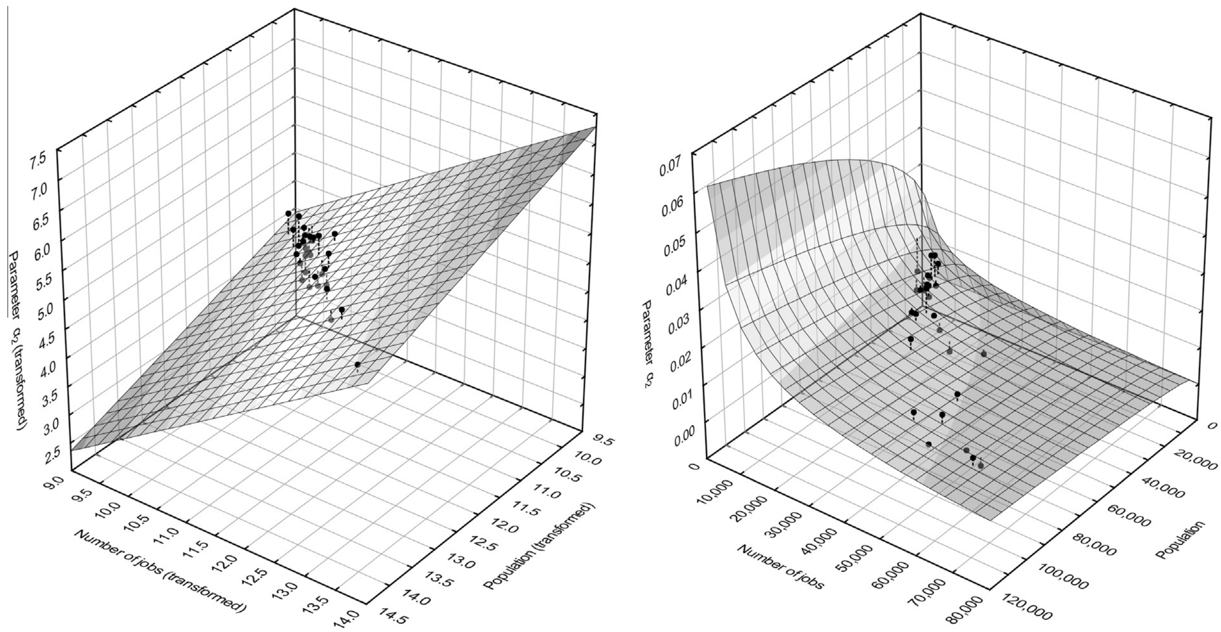

Fig. 14. Bivariate regression model of the dependence of the α_2 parameter on population and number of jobs.

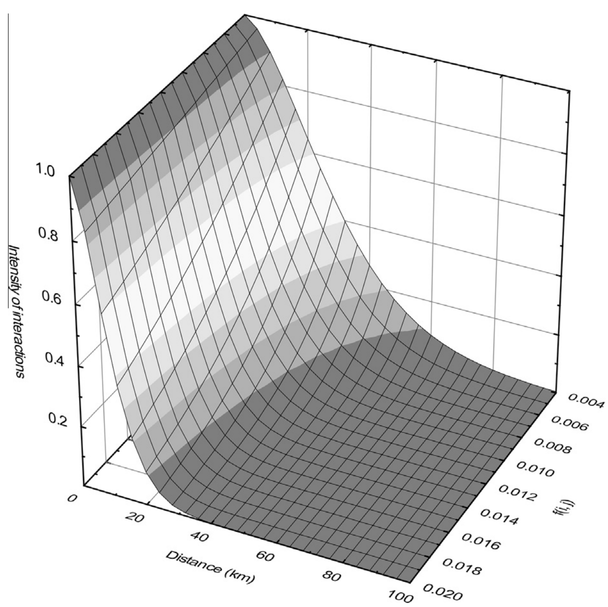

Fig. 15. Universal distance-decay function for daily travel-to-work to Czech regional centres (dependent on population and number of jobs).

parameters, the figure would become illegible if it included all regional centres).

On the basis of the preceding computations the universal distance-decay function for regional centres can be expressed by the dependency on the population and number of jobs. The optimal function presenting the changes in interaction intensities dependant on the distance from regional centre, population and number of jobs is[3]:

$$f(d,i,j) = exp(-0.4334 \cdot i^{0.3132} \cdot j^{-0.6738} \cdot d^{1.57}).$$

[3] Procedure is analogical to footnote 1.

This function is presented in Fig. 15; while it is a function of three variables, which can be plotted only in four dimensions, the y axis is presented as a function of the population and number of jobs: $f(i,j) = 0.4334.\ i^{0.3132} \cdot j^{-0.6738}$. For our sample of regional centres the function $f(i,j)$ takes the values from the interval (0.004;0.019). While the α_2 parameter correlates negatively to the size of a centre (expressed either by the population or number of jobs) the values along the y axis are reversely ordered. The shape of the curve and radius of influence S change differently from the univariate dependency. For the univariate dependency the size of the centre increases linearly, while for the bivariate dependency the value of function $f(i,j)$ changes linearly, not the size of the centre itself (Fig. 15).

6. Conclusions

Although geographical space is considerably heterogeneous and daily movements of the population into regional centres are influenced by a number of more or less relevant factors, it is possible to quantify these movements and the changes in their intensities. The application of this method in the case of the Czech Republic has shown that such quantification has relatively great information value. The distance-decay function, together with a detailed analysis of its shape, considerably helps our purpose.

For the approximation of daily labour commuting into regional centres, the bell-shaped functions – i.e. the decreasing function with the inflection point, changing its curve from concave to convex – is the most suitable. The analyses have been carried out for virtually all centres in the Czech Republic (therefore there are many examples in the paper) in order to acquire maximum possible data for estimation of the universal distance-decay function. Three points should be highlighted here: (1) Apart from minor exceptions (e.g., Ústí nad Labem) the distance-decay function shows very similar behaviour for most regional centres in the Czech Republic and differs substantially only as to the extent of regional influence. (2) For the spatial expression of the regional influence a new parameter has been introduced, the so- called radius of influence *S* which is the area below the function curve. (3) The similarity of the behaviour of the functions and the dependency of the *S* parameter on the characteristic of regional centre size made it possible to also express a so-called universal distance-decay function depending on the selected characteristic. This method is universal, but this universality somewhat reduces its information value.

What can be seen if we take a closer look at the distance-decay functions and their parameters with respect to the Czech settlement system and general characteristics of geographical environment of the Czech Republic? More information is provided by the analysis basic (individual) distance-decay functions calibrated for each regional centre in this respect. The dominant position of Prague and its regular coexistence with large centres (Plzeň, České Budějovice, Liberec – see Fig. 1 for general orientation for the findings presented in the following paragraphs) is asserted in Bohemia. The dominance of Brno in Moravia is a little lower; more equal relations are typical for adequate centres in the Moravian–Silesian space. This is caused by the lower hierarchical position of Brno in comparison to Prague and by the more even relations, in terms of size, accessibility, and distribution of the communication network, between Brno and other Moravian–Silesian centres.

Some smaller centres possessing a massive economic base and producing large commuting hinterlands (both in terms of population and area) also have a strong position (particularly Mladá Boleslav). The radius of influence is larger (relative to their size) in the case of centres without an adequate competitor in their vicinity; however, these hinterlands are larger only in terms of area and not of population (e.g. České Budějovice, Jihlava, Písek, etc.). We can speak similarly of Znojmo, but thanks to its location 10 km from the Austrian border, its hinterland does not actually comply with the radius of influence *S* as seen in Table 1.

The relative variability of the values of the coefficient of determination (Table 1) reflects the heterogeneity of the geographical space as was hinted at in the introduction. The rule here appears to apply as follows. The more geographical barriers found in the surroundings of a regional centre, the lower the value of the coefficient of determination. High values for the coefficient of determination indicate that the distance-decay function expresses relatively well the daily movements of the population into the regional centres. Generally, higher values of the coefficient are found for centres dominating their surroundings (Brno, České Budějovice, Jihlava).

Lower values of the coefficient of determination indicate the centres with deformed potential hinterlands. These deformations are conditioned by several different factors that sometimes act mutually and that determine the spatial permeability for interaction in various directions. The influence of physical geographical barriers is witnessed in the cases of Liberec, Vsetín, Česká Lípa, Trutnov and Karlovy Vary. Political factors are documented by three examples. (1) International borders play their role in the cases of Znojmo, Třinec and Krnov. (2) Existence of military areas is responsible for lower values of the coefficient of determination in the cases of Karlovy Vary and Česká Lípa (here we can see the multiplying effects of the barrier factors). (3) Administrative division affected the position of Uherské Hradiště and its value of the coefficient of determination. In fact there is an agglomeration of three towns that used to form one administrative unit and still are functionally tied together, but only Uherské Hradiště qualifies as a regional centre. The character of the settlement system is responsible for the lower values of the coefficient of determination for regional centres which have in their vicinity centres several times larger and more economically powerful (e.g., Kladno and Kolín near Prague, Třinec and Frýdek-Místek near Ostrava, or Kroměříž near Zlín).

The estimation of the universal distance-decay function opens new opportunities for research. As it is a compound power-exponential function with the inflexion point, it can react more flexibly to some changes in the character of interactions and to the changes in the size of population and job position, independently of the population censuses. It could also be applied to different territories. Further research into the distance-decay function for the regional centres of the Czech Republic can be aimed at their practical spatial applications, such as communication network planning analysis or the identification of boundaries of regional influence of centres and subsequent regionalisation tasks. The universal distance-decay function may also enter the spatial interaction models that can provide more precise results reflecting objective reality.

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